AFIT/GOR/ENS/95M-13

RIGHT-HAND-SIDE MULTIDIMENSIONAL OPTIMALITY ANALYSIS OF A LARGE SCALE LINEAR PROGRAM USING METAMODELLING TECHNIQUES

THESIS

Osman İYDE
First Lieutenant

AFIT/GOR/ENS/95M-13

The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U. S. Government.

AFIT/GOR/ENS/95M-13

RIGHT-HAND-SIDE MULTIDIMENSIONAL OPTIMALITY ANALYSIS OF A LARGE SCALE LINEAR PROGRAM USING METAMODELLING TECHNIQUES

THESIS

Presented to the Faculty of the Graduate School of Engineering
of the Air Force Institute of Technology
Air Education and Training Command
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Operations Research

Osman İYDE, B.S.
First Lieutenant

March, 1995

THESIS APPROVAL

STUDENT : 1Lt Osman İYDE CLASS : GOR-95M

THESIS TITLE : RIGHT-HAND-SIDE MULTIDIMENSIONAL OPTIMALITY ANALYSIS OF A LARGE SCALE LINEAR PROGRAM USING METAMODELLING TECHNIQUES

DEFENSE DATE : 23 February 1995

COMMITEE :	NAME/DEPARTMENT	SIGNATURE
Advisor	LtCol James T. Moore, Ph. D. /ENS Associate Professor of Operations Research	James T. Moore
Reader	LtCol Kenneth W. Bauer, Jr. Ph. D. /ENS Associate Professor of Operations Research	Kenneth W Bauer, Jr.

Preface

This thesis employs the application of the methodology developed by Johnson, Bauer, Moore, and Grant to a large scale linear programming (LP) model, specifically AMC's STORM. The methodology of Johnson, *et al.* utilizes response surface methology and kriging. It basically accomplishes an optimality analysis for LP models by developing first or second order metamodels which describe the relationships between the optimal objective function value and the RHS vectors of the LPs.

This research may benefit the analysts of AMC because the developed metamodels can provide some valuable insights about the system when the resources of the channel cargo system are changed.

My Special thanks go to my advisor LtCol James T. Moore and my thesis reader LtCol Kenneth W. Bauer for their invaluable guidance and support. Without them, the existence of this thesis effort would be impossible.

This research was sponsored by Air Mobility Command. I wish to thank Mr. Alan Whisman for his help on providing the data and information about the channel cargo system of AMC.

I would also like to thank my classmates, especially Güneş and Duman. Duman's assistance on LaTeX saved me great time.

Most of all, I thank my girlfriend Leslie Ann for her patience and understanding while helping me improve my English.

Osman İYDE

Table of Contents

Page

List of Figures

List of Tables

AFIT/GOR/ENS/95M-13

Abstract

A methodology for optimality analysis of linear programs was developed by Johnson, Bauer, Moore, and Grant to create metamodels using response surface methodology techniques such as experimental design and least squares regression, and a geostatistical estimation technique, namely kriging. Metamodels have the form of a simple polynomial, and they predict the optimal objective function value of an LP for various levels of the constraints. They eliminate the necessity of determining which critical region contains the right-hand-side (RHS) vector of interest since they are valid over multiple critical regions.

The methodology of Johnson, *et al.* can be applied to large scale linear programming models. The developed metamodels of the large scale LP can provide some useful information about the relationships between the objective function value and the RHS vector of interest.

RIGHT-HAND-SIDE MULTIDIMENSIONAL OPTIMALITY ANALYSIS OF A LARGE SCALE LINEAR PROGRAM USING METAMODELLING TECHNIQUES

I. Introduction

This research supports the efforts of Johnson, Bauer, Moore, and Grant (21), and investigates the application of the methodology they have developed to a large scale linear program.

The methodology uses experimental design, linear regression, and kriging techniques to perform an optimality analysis. This study is sponsored by the United States Air Force Air Mobility Command (AMC). The first chapter provides the background, the research objective, the scope of the study, and a summary of current knowledge.

1.1 Background

Linear programming (LP) is a tool for solving optimization problems. Since the development of the simplex algorithm for solving LPs by George Dantzig in 1947, and later the development of interior point methods, LP has been used to solve the optimization problems of industries and government. In a survey by Fortune magazine, 85 percent of 500 responding firms indicated that they used linear programming for optimization purposes (35:47).

It is difficult to give a precise definition of a "large" LP model since the definition changes according to the user and the tools used for solving and analyzing these large models. Brook, Kendrick, and Meeraus (9:166-167) describe large models as follows:

> What is a large model? The answer varies. It is one that takes a lot of time (or money) to solve. It is one that "just fits" into memory available on your machine. It is one that has more than a few hundred lines of assignment and equations when written in GAMS. Briefly, any model that is expensive to solve or difficult to manage, or whose details are so overwhelming that it hard to keep track of them is large.

Approaches to solving large scale LPs were initiated with the publication of the Dantzig-Wolfe decomposition principle in 1960. However, there was significant work in this area even before that. Since then, the growth of studies has been explosive (26:144). As the size of LPs increased, the difficulty of optimality analysis, as described below, has increased. Although, today we are able to solve large LPs with millions of variables and hundred thousands of constraints by using highly developed computer packages, we still need efficient optimality analysis methods.

The right-hand-side (RHS) vectors of an LP may be changed for a variety of reasons: we may want to conduct a sensitivity analysis to check the preciseness of the RHS vectors, and whether or not it matters if the RHS vectors are perturbed. Also, we may want to update the LP when additional (reduced) resources become available (unavailable). In this case, optimality analysis comes into play. Optimality analysis is performed to determine the effect on the optimal solution when the right-hand-side vector (or the objective function coefficient) is changed. Optimality analysis generally involves multiple critical regions with different optimal bases. This differs from "post-optimality analysis" and "sensitivity analysis" since those analyses deal with only one critical region (21). Optimality analysis of large scale LPs across multiple critical regions is more difficult than situations dealing with only one critical region. Thus, identifying which critical region contains a particular right-hand-side vector creates a burden for the analyst.

Unlike other multiparametric techniques developed by Gal and Nedoma (14) Mulvey, *et al.* (28) and Wagner (32), the methodology proposed by Johnson, *et al.* creates metamodels by using experimental design, linear regression, and kriging to perform optimality analysis. This methodology eliminates the necessity of determin-

ing which critical region contains a right-hand-side vector of interest. Furthermore, it insures efficient data requirements, and it allows us to understand the key relationships between the objective function and the constraints. The method develops metamodels that can provide valuable insights about the system. Sometimes the metamodels are able to represent the whole system (21).

1.2 Research Objectives

We will employ a large scale LP where the right-hand-side (RHS) vectors can vary over a range of values. A multidimensional metamodelling technique for estimating the objective function over the right-hand-side vector has been developed by Johnson, *et al.* The problem is to verify whether or not this technique is valid for large scale linear programs. Since metamodels are really time and effort savers, the analyst will be able to observe the response of the optimal objective function value of a particular LP very easily and efficiently when the levels of the constraints of this LP are changed.

1.3 Scope of the Research

This research performs optimality analysis only on the right-hand-side vector. We intend to create metamodels with only first order polynomials unless the higher order polynomials (such as second order) are needed. We basically apply 2^{k-p} fractional and 2^k full fractional designs to create metamodels by least squares regression. Furthermore, in this research we apply only ordinary kriging techniques (point kriging and block kriging) assuming that there is no drift in the data set.

For the evaluation of metamodels, three primary measures are used: mean squared error (MSE), percentage of predictions closer to the true optimal solution (PC), and mean absolute percentage error (MAPE).

1.4 Summary of the Current Knowledge

In this thesis, STORM was chosen as a large scale LP for the application of the methodology of Johnson *et al.* This section first introduces STORM, and then gives a brief summary of the approaches to sensitivity analysis.

1.4.1 Strategic Transport Optimal Routing Model (STORM) . The Strategic Transport Optimal Routing Model (STORM) is based on a model built by Barton and Gumaer (1967) for Lockheed to analyze the peacetime employment of the new C-5 cargo plane. STORM was developed at the Air Mobility Command (then the Military Airlift Command) to assist in a major study of the entire scheduled cargo system that must provide two main types of service to its overseas customers. The first is to provide sufficient cargo capacity for a given period of time (usually for one month) to meet all demands for cargo movement between the pairs of cities in the system. This cargo capacity is known as the cargo requirement. The second is to provide a minimum number of flights per month between certain cities. This number is called the frequency requirement. The basic purpose of STORM "is to select the mix of routes and aircraft that will meet the monthly cargo and frequency requirements while minimizing the cost of cargo handling, military aircraft operations, and commercial aircraft leasing" (1:2).

The set of routes, maximum payload, and total flying hours for each type of plane are the main resources available from which STORM constructs a feasible cargo movement plan. Routes are the sequences of legs to be flown by a single aircraft. Each aircraft's maximum payload is an average based on the fuel loads and types of palletized cargo that are generally carried on the planned missions. The flying hour limit for military aircraft is derived from the Air Force flying program which is necessary to maintain proficiency in worldwide operations and to train the crews.

Versions of STORM for UNIX Workstations have been developed using the GAMS modelling language (9) which makes data management and programming

very easy. GAMS also allows the analyst to modify the model quickly for specific analyses, investigating specific questions, or enforcing operational considerations locally. For detailed information about the modelling of STORM using GAMS, refer to Appendix A.

In the next sections we divide the sensitivity analysis approaches into two cases according to the critical regions they are dealing with.

1.4.2 The Case When the Candidate Set is In a Single Critical Region. If we do not want to change the optimal basis, or we are certain that potential changes in a set of RHS vectors will occur in a single region, then we can use one of the techniques developed for multidimensional sensitivity analysis. Some examples of these techniques are summarized below.

Higher Dimensional Cuts. A cut is a line which divides the critical region of the candidate set into two parts. Making a one-dimensional cut through the critical region of the RHS vector and characterizing the end points of this cut was proposed by Gass in 1954. One-at-a-time restriction is a major limitation of this approach (33:14). The RHS vectors often vary simultaneously and dependently so that one-dimensional cuts are not applicable. Rather than making a one-dimensional cut of a critical region, another approach to simultaneous perturbations is to make higher dimensional cuts (*see* Gass and Saaty p.26) (15).

100 Percent Rule. As proposed by Bradley, Hax and Magnanti (8:7), one-dimensional bounds can be used indirectly to give an approximation to a critical region by using a procedure called the 100 Percent Rule. This rule requires a specification of the increase or decrease in the direction for each RHS vector. This guarantees that the same basis remains optimal as long as the sum of the fractions which correspond to the percent of maximum change in each direction is less than or equal to one.

The Tolerance approach. This approach was described by Wendell (34). In contrast to "ordinary" sensitivity analysis in linear programming, it considers simultaneous and independent changes in the objective function coefficients and in the RHS vectors. The tolerance approach yields a maximum tolerance percentage such that, as long as selected coefficients or vectors are accurate within that percentage of their estimated values, the same basis remains optimal.

A Perturbation Analysis Approach by Arsham and Oblak (3). This technique allows simultaneous perturbation of the RHS vector and/or the cost coefficients and/or the matrix coefficients from their nominal values. Using the perturbed optimal solution, the current optimal solution is updated for various points within the critical region.

Convex Bounds. The adaptation of the exploitation of convex bounds to estimate the effects of large perturbations of the RHS vectors or objective function coefficients in an LP was derived from the properties of convexity by Fiacco (13). He describes iterative procedures for choosing additional points to improve these bounds. This boundary approach helps to address the computational difficulties of computing the optimal objective function value for higher dimensional cuts.

1.4.3 Approaches Dealing With Multiple Critical Regions. If the set of RHS vectors contains vectors from more than one critical region, analysis of an LP becomes more difficult. Techniques developed for this analysis are summarized below.

Gal and Nedoma's Methodology for Multiparametric Programming. In their paper, Gal and Nedoma (14) present an effective method for finding all non-overlapping critical regions that cover a prespecified admissable set. This method uses an algorithm to determine which critical region contains a particular RHS vector. The algorithm is developed so that it uses an iterative approach and utilizes one of the techniques described in section 1.4.1, for example, the tolerance approach of Wendell.

The Methodology of Mulvey, Vanderbei, and Zenios (28). This methodology finds robust solutions to optimization problems. Mulvey, *et al.* use various scenarios to identify the uncertainty in model parameters. They employ a multi-objective programming framework to resolve conflicting objectives. The solution to the problem is "solution robust" if it remains close to optimal across various scenarios. In contrast, a solution is "model robust" if it remains "almost" feasible across the scenarios applied. Basically Mulvey, *et al.* seek a simple robust solution in reaction to noisy parameters (21).

Methodology of Wagner (32). Wagner uses Monte Carlo methods in his methodology, and assumes knowledge of the joint probability distribution of a vector of parameters of interest. He takes samples from this distribution, solves the resulting optimization problem, and produces a data set that serves as an input to a regression routine. Wagner basically seeks a characterization of the optimal solution as a function of parameters which vary according to a prespecified probability distribution (21:15). The methodology proposed by Johnson, Bauer, Moore, and Grant can be considered similar to Wagner's methodology in purpose. However, the methodology suggested by Johnson, *et al.* uses response surface methods (specifically experimental design and least squares regression) and kriging instead of joint probability distributions for Monte Carlo methods. Johnson, *et al.* develops metamodels whose form is a polynomial describing the optimal objective function value as a function of the RHS vector.

II. Literature Search and Review

This chapter discusses some basic aspects of linear programming and the techniques used in the methodology of Johnson, *et al.* which we will apply to a large scale system, specifically AMC's STORM model. These techniques include experimental design, least squares regression, and kriging. Other approaches to optimality analysis were summarized in Section 1.4 of Chapter 1.

2.1 Linear Programming

The general linear program is described by Hadley (17:4) as follows:

> Given a set of m linear inequalities or equations in r variables, we wish to find non-negative values of these variables which will satisfy the constraints and maximize or minimize some linear function of the variables.

To represent an optimization problem as a linear program, assumptions about proportionality, additivity, divisibility and deterministic coefficients are made implicitly during formulation (4:3-4). With the inclusion of slack variables and simple transformations, an LP can be stated in matrix form as:

$$\begin{aligned} \text{Minimize} \quad & \mathbf{cx} \\ \text{Subject to} \quad \mathbf{Ax} &= \mathbf{b} \\ \mathbf{x} &\geq \mathbf{0} \end{aligned} \tag{2.1}$$

where $\mathbf{c}$ is a $1 \times n$ vector of objective function coefficients, $\mathbf{x}$ is an $n \times 1$ vector of decision and slack variables, $\mathbf{A}$ is an $m \times n$ matrix of the constraint coefficients, and $\mathbf{b}$ is an $m \times 1$ vector of the right-hand-sides of the constraints.

2.1.1 Existence of Solutions. For any right-hand-side vector, $\mathbf{b} \in \Re^m$, four possible cases may arise for the linear program.

1. *Unique Finite Optimal Solution.* When an optimal finite solution is unique, it occurs at an extreme point. Figures *2.1a* and *2.1b* show a unique optimal solution. In Figure *2.1a*, the feasible region is bounded. In Figure *2.1b*, the feasible region is not bounded.

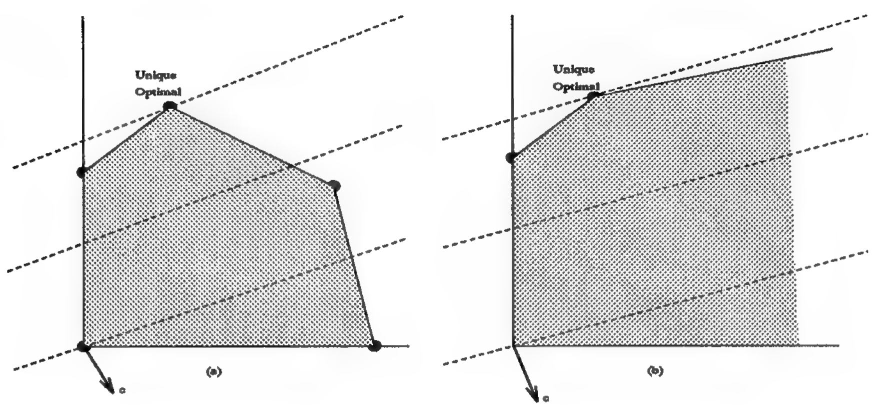

Figure 2.1 Unique finite optimal solution: (a) Bounded region. (b) Unbounded region.

2. *Alternate Finite Optimal Solutions.* Figure *2.2a* shows this case when the feasible region is bounded. The two extreme points, x_1^* and x_2^*, are optimal, as is any point on the line segment joining them. In Figure *2.2b*, the feasible region is unbounded but the value of the objective function at optimality is finite.

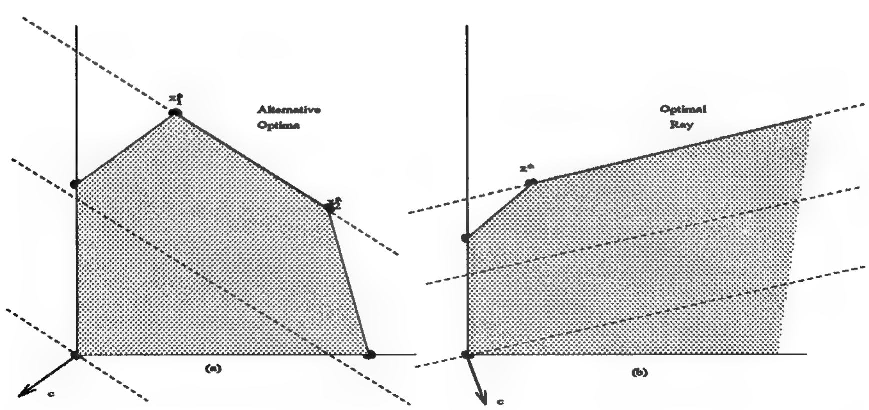

Figure 2.2 Alternate finite optima: (a) Bounded region. (b) Unbounded region.

3. *Unbounded Optimal Solution Value.* This case is illustrated in Figure *2.3* where both the feasible region and the optimal solution value are unbounded. Any point on the "ray" with vertex x^* is feasible. So, the value of the solution is unbounded. No optimal solution exists.

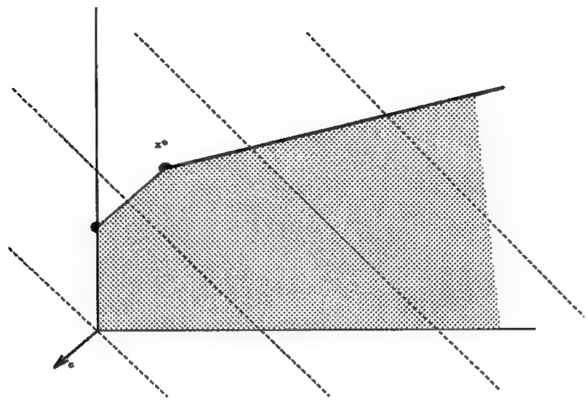

Figure 2.3   Unbounded optimal solution.

4. *Empty Feasible Region.* In this case, the system of equations and/or inequalities defining the feasible region is inconsistent (4:18). In Figure *2.4*, it is clear that there is no point (x_1, x_2) satisfying the inequalities.

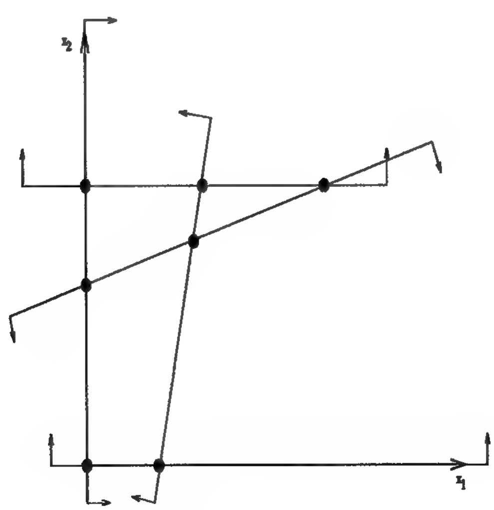

Figure 2.4   Empty feasible region.

2.1.2 Interpretation of Feasibility. Consider the linear programming problem given by (2.1). For the $m \times n$ **A** matrix, the jth column is denoted by $\mathbf{a_j}$. The problem can be written as follows:

$$
\begin{array}{llcl}
\text{Minimize} & \sum_{j=1}^{n} c_j x_j & & \\
\text{Subject to} & \sum_{j=1}^{n} \mathbf{a}_j x_j & = & \mathbf{b} \\
 & \mathrm{x}_j & \geq & 0 \quad j = 1, 2, ..., n
\end{array}
\qquad (2.2)
$$

Given the vectors $\mathbf{a}_1, \mathbf{a}_2, .., \mathbf{a}_n$, we are trying to find nonnegative scalars $x_1, x_2, ..., x_n$ such that $\sum_{j=1}^{n} c_j x_j$ is minimized, and $\sum_{j=1}^{n} \mathbf{a}_j x_j = \mathbf{b}$. However, note that the collection of vectors of the form $\sum_{j=1}^{n} \mathbf{a}_j x_j$, where $x_1, x_2, ..., x_n \geq 0$, is the cone generated by $\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_n$ (4:20). This cone is also called *the positive cone of* **A**, and is denoted by *POS (A)* (29). The problem has a feasible solution if and only if the vector $\mathbf{b} \in POS(A)$ (see Figure *2.5a* and *2.5b*). Because the vector **b** reflects some requirements to be satisfied, Bazaraa, *et al.* call Figure *2.5* a requirement space (4:20).

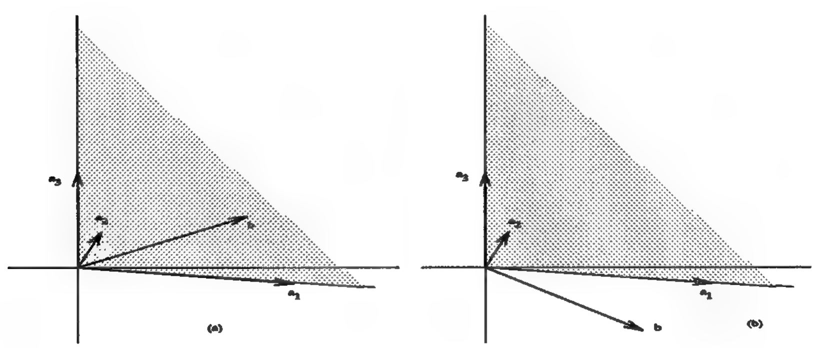

Figure 2.5 Illustration of the requirement space: (a) System is feasible. (b) System is inconsistent.

The requirement space and inequality constraints are illustrated in Figure *2.6*. If a feasible solution exists, then the cone generated by $\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_n$ must intersect the collection of vectors that are less than or equal to the requirement vector **b**.

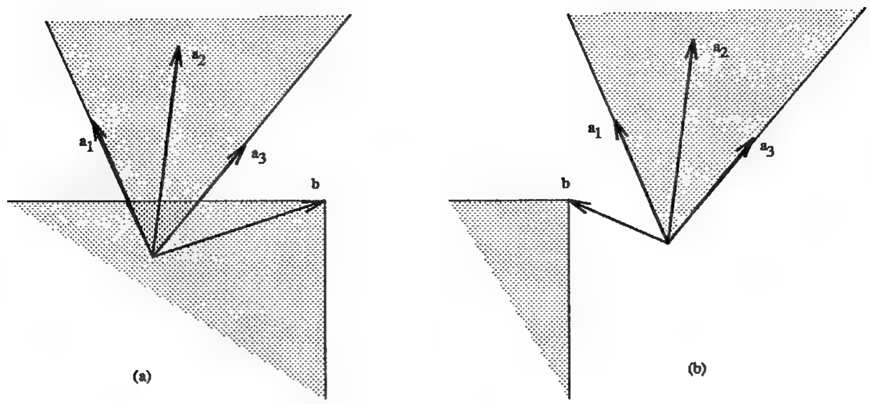

Figure 2.6 Requirement space and inequality constraints: (a) System 1 is feasible. (b) System 2 is infeasible.

2.1.3 Optimality. Given a basic feasible solution $\mathbf{x_B} = \mathbf{B^{-1}b}$, where $\mathbf{B}$ is a nonsingular $m \times m$ submatrix of $\mathbf{A}$, with $z_0 = \mathbf{c_B x_B}$ for the LP $\mathbf{Ax} = \mathbf{b}$, $\mathbf{x} \geq \mathbf{0}$, *min* $z = \mathbf{cx}$. If $z_j - c_j \leq 0$ for every column $\mathbf{a}_j$ in $\mathbf{A}$ where $z_j = \mathbf{c}_B\mathbf{B}^{-1}\mathbf{a}_j$, then z_0 is the minimum value of z subject to the constraints, and the basic feasible solution is an optimal basic feasible solution (17:97).

2.1.4 Critical Regions. The set of RHS vectors satisfying the feasibility condition is called the critical region for $\mathbf{B}$ where $\mathbf{B}$ is an optimal basis for all RHS vectors $\mathbf{b}$ such that $\mathbf{B}^{-1}\mathbf{b} \geq \mathbf{0}$. Each critical region is a convex cone, the number of critical regions associated with an LP problem is finite, and *POS(A)* is the union of the critical regions (21:11).

2.1.5 Objective Function Value as a Function of the RHS Vector. Let $\mathbf{\Phi}(\mathbf{b})$ denote the optimal objective function value as a function of the RHS vector $\mathbf{b}$. For each $\mathbf{b}$, either 1) there exists an optimal basis $\mathbf{B}^i$ for every critical region $i = 1, ..., n$ such that $\mathbf{\Phi}(\mathbf{b}) = \mathbf{c_B x_B} = \mathbf{c_B}[\mathbf{B}^i]^{-1}\mathbf{b}$ or 2) $\mathbf{b} \notin POS(A)$ and the feasible region is empty (21:15). $\mathbf{\Phi}$ is piecewise linear over *POS(A)*. Since $\mathbf{\Phi}$ is a continuous and concave function over *POS(A)* (see Murty, *Theorems 8.4* and *8.5*, p. 289) (29), by

the Weierstrauss Approximation Theorem, $\mathbf{\Phi}$ can be approximated by a polynomial (2:481).

2.2 Response Surface Methodology

Response surface methodology (RSM) was introduced by G.E.P. Box and K. B. Wilson in 1951 (7). It was developed for the purpose of approximating a surface in a region of interest in order to search for an optimal value.

RSM comprises a set of statistical and mathematical techniques for empirical model building and exploitation that encompasses 1) Designing a series of experiments that will yield adequate and reliable measurements of the response(s); 2) Analyzing the results of those experiments to determine a mathematical model that best fits the data collected, and 3) Searching for the optimal settings of the input variables that produce a desired response (25).

To locate the combination of factors which provide the extreme value on an implicit or explicit response surface, direct methods and approximation methods are used. The direct methods consist of direct elimination techniques, hill climbing techniques, and the single-factor method. In contrast to direct methods which use a sequential method, approximating methods consist of a single large experiment. The data points are chosen randomly or by the factorial approach of selecting all possible combinations of the factor levels. Then, the response is evaluated at each of these points in order to approximate the true functional relationship between the response and the factors. This approximation is usually developed by fitting a polynomial to the data. Then, this polynomial is manipulated by classical methods to approximate the true optimal response (31:29).

2.2.1 Experimental Design. Box (5:31) defines an experimental design as "Each combination of levels of the factors corresponding to a point in the factor

space and the pattern of such points used to elucidate the [response] surface is called the experimental design."

Montgomery (27:27) defines experimental design as "a process of inducing purposeful changes in the input variable(s) in order to observe and model the changes in the response(s)."

The general problem of experimental design in response surface methodology is to choose a design such that the assumed polynomial that is fitted by the least squares regression most closely represents the true function over the factor space of interest. Experimental designs are selected so that the experimental error is minimized or the bias due to the model misspecification is minimized.

Today, a class of experimental designs known as factorial designs are widely used. Factorial designs; 1) allow many simultaneous comparisons to be made; 2) yield highly efficient parameter estimates; and 3) are computationally simple to analyze (6:106). A factor is a variable or characteristic that is changed during the experimentation. Factors may have different levels which are the values to be set during the course of experimentation. A specified combination of levels for all factors included is called a design point. So, an experimental design can also be defined as a schedule of the design points to be investigated in an experiment.

Experimental designs are usually divided into classes according to the degree (order) of polynomial to be produced. Examples are first order designs and second order designs. Furthermore, factorial designs can be grouped according to the design points considered. A complete (or full) factorial design is a design containing every possible combination of the levels of all factors. If there are k factors with each factor $i = 1, 2, ..., k$ having n_i levels, then the resulting full factorial design n^k contains $n_1 \times n_2 \times ... \times n_k$ design points; if $n_1 = n_2 = ... = n_k = n$, then it is referred as an n^k design. For example, 2^4 indicates that it is a two level full factorial design on four factors (variables) and 16 design points (trials).

Fractional factorial designs are designs which require only a fraction of the number of trials required by a full factorial design in order to provide the minimal information requirements (31:37). For example, a 2^{k-p} design is a $\frac{1}{2^p}$ fraction of a 2^k design. A half replicate is a one-half fraction or a 2^{k-1} design; a quarter replicate is a one-fourth fraction or a 2^{k-2} design. Although fractional factorial designs are very practical in that they save money and effort for large experiments, the main disadvantage of their use is that there is some confounding of the effects. Confounding means that "the statistic which measures a main effect will be identical to the statistic that measures certain other interaction effects" (19:426). For example, many fractional factorial designs are used to measure main and two-factor interaction effects, and it is assumed that the higher-order effects with which the main effects are confounded are very small.

Resolution is the length of the shortest effect in the defining relation in a two level fractional design. Let us denote the factor by *f*. If a 2^{k-p} design is of resolution R, then no *f* effect will be confounded with any other effect containing less than R-f factors. For example, for a resolution IV fractional factorial design of 2^{k-p}, no main effects are aliased with any other main effects or with any two-factor interactions; however, two-factor interactions are aliased with each other.

2.2.2 Regression Analysis. Regression analysis was first developed by Sir Francis Galton in the latter part of the 19th Century. Galton had studied the relation between the height of fathers and sons and noted that the heights of sons of both tall and short fathers appear to "revert" or "regress" to the mean of the group. He considered this tendency to be a regression to "mediocrity". Galton developed a mathematical description of this regression tendency which is the ancestor of today's regression models (30).

Neter, *et al.* (30) summarizes the basic concept as:

> A regression model is a formal means of expressing the two essential ingredients of a statistical relation: 1) A tendency of the dependent vari-

able Y to vary with the independent variable in a systematic fashion. 2) A scattering of points around the curve of statistical relationship. These two characteristics are embodied in a regression model by postulating that: 1) There is a probability distribution of Y for each level of X. 2) The means of these probability distributions vary in some systematic fashion with X.

The functional relationship between the responses and the variables can be defined as

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \tag{2.3}$$

where $\mathbf{Y}$ is $s \times 1$ vector of responses, $\mathbf{X}$ is a $s \times q$ matrix of the coded levels of the variables, $\boldsymbol{\beta}$ is a $q \times 1$ vector of parameters, $\boldsymbol{\epsilon}$ is a $s \times 1$ vector of independent normal random variables with expectation $\mathbf{E}\{\boldsymbol{\epsilon}\} = \mathbf{0}$, s is the number of data points, and q is the number of variables.

The least square estimators of the regression model are

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y} \tag{2.4}$$

These least squares estimators are also "maximum likelihood estimators", and they are "unbiased, minimum variance unbiased, consistent, and sufficient" (30:238).

The fitted values and residual terms are represented by

$$\hat{\mathbf{Y}} = \mathbf{X}\hat{\boldsymbol{\beta}} \tag{2.5}$$

and

$$\hat{\boldsymbol{\epsilon}} = \mathbf{Y} - \hat{\mathbf{Y}} = \mathbf{Y} - \mathbf{X}\hat{\boldsymbol{\beta}} \tag{2.6}$$

where $\hat{\mathbf{Y}}$ is the $s \times 1$ vector of fitted values, and $\hat{\boldsymbol{\epsilon}}$ is a $s \times 1$ vector of residuals.

The goal of least squares regression is to determine an estimate $\hat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$ so that the residual sum of squares $\hat{\boldsymbol{\epsilon}}'\hat{\boldsymbol{\epsilon}}$ is minimized. Generally, $\boldsymbol{\epsilon}$, the residuals, are assumed to be caused by random measurement error (21:20).

2.3 Kriging

Kriging is a geostatistical estimation technique developed by Matheron in the 1960's to estimate ore reserves from spatial core samples. It is named after D.G. Krige who was probably the first to make use of spatial correlation and best linear unbiased estimator (B.L.U.E.) in the field of mineral recourses evaluation (11:237).

Journel describes kriging as "a local estimation technique which provides the best linear unbiased estimator of the unknown characteristics studied" (23:304). So, kriging is associated with "B.L.U.E.". Kriging is "best" because it minimizes the variance of the errors; it is "linear" because its estimates are weighted linear combinations of the available data; it is "unbiased" since it tries to have the mean residual or error equal to zero. The distinguishing feature of kriging from other estimation methods is its aim of minimizing the error variance (20:278).

2.3.1 Kriging Equations. In kriging, the value of an unknown point is estimated using a weighted average of the values of previously sampled points in such a way that the points close to the point that we are trying to estimate must have more weight then the ones far away (11).

The estimator is given in matrix form by

$$\hat{\mathbf{Y}}_{\mathbf{p}} = \mathbf{W}'\mathbf{Y} \tag{2.7}$$

where $\hat{\mathbf{Y}}$ is the estimate, $\mathbf{W}$ is the $s \times 1$ vector of the weights, and $\mathbf{Y}$ is the $s \times 1$ vector of the values for the known sample points.

To determine the combination which minimizes the estimation error, the estimation variance must be defined. The estimation variance for the general unbiased linear estimator is

$$\sigma^2 = 2\sum_{i=1}^{s} \omega_i \bar{\gamma}(S_i, \hat{Y}_p) - \sum_{i=1}^{s}\sum_{j=1}^{s} \omega_i \omega_j \bar{\gamma}(S_i, S_j) - \bar{\gamma}(\hat{Y}_p, \hat{Y}_p) \tag{2.8}$$

where σ^2 is the estimator variance, ω_i and ω_j are the weights, s is the size of the sample set, $\hat{Y}$ is the point to be estimated and $\omega_i \bar{\gamma}(S_i, \hat{Y}_p)$ is known as the average semi-variogram. Clark describes a semi-variogram as "a graph (and/or formula) describing the expected difference in value between pairs of samples with a given relative orientation" (10:11).

If the weights sum to one and there is no drift (no trend), the estimates are considered unbiased (12:384). The weights are found by using the following kriging equations in matrix form:

$$\begin{bmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & \gamma_{h_{11}} & \gamma_{h_{12}} & \cdots & \gamma_{h_{1s}} \\ 1 & \gamma_{h_{21}} & \gamma_{h_{22}} & \cdots & \gamma_{h_{2s}} \\ \vdots & \vdots & \ddots & \vdots & \\ 1 & \gamma_{h_{s1}} & \gamma_{h_{s2}} & \cdots & \gamma_{h_{ss}} \end{bmatrix} \begin{bmatrix} \lambda \\ \omega_1 \\ \omega_2 \\ \vdots \\ \omega_s \end{bmatrix} = \begin{bmatrix} 1 \\ \gamma_{h_{1p}} \\ \gamma_{h_{2p}} \\ \vdots \\ \gamma_{h_{sp}} \end{bmatrix} \tag{2.9}$$

where $\omega_{h_{ij}}$ are the semi-variograms between the points in the neighborhood, $\gamma_{h_{ip}}$ are calculated semi-veriograms between the points in the neighborhood and the estimated point, λ is a Lagrange multiplier used to solve the simultaneous equations, and h_{ij} are the distances between sampled points.

2.3.2 Types of Kriging. Although there are several types of kriging such as lognormal, disjunctive, point, block, universal, and positive kriging, they are all related, and they are the refined versions of the weighted moving average techniques (18:25).

Point kriging (ordinary kriging). This kriging technique is used when we assume that there is no trend or drift in data. To calculate the point kriging weights, the pattern of spatial continuity that one desires the random function model to obtain must be decided. Random function models are produced by variograms. Some standard models are generated for variograms. The smallest standard model is a linear model of the form $\gamma(h) = ah + b$, which does not have a sill variance of the samples. Probably the most common standard model is the spherical model which uses the parameters a, C, and C_0. The first parameter a is called range, and it provides a distance beyond which the variogram or covariance value remains essentially constant. C_0 is called the nugget effect, and it provides a discontinuity at the origin. C is used in conjunction with C_0 to determine the sill, $(C + C_0)$. The sill is the variogram value for very large distances, $\gamma(\infty)$. It is also the covariance value for $\mid h \mid$ (20:293). The form of the spherical model is given by

$$\gamma(h) = \begin{cases} C(\frac{3h}{2a} - \frac{h^3}{2a^3}) + C_0 & \text{if } h < a \\ C + C_0 & \text{if } h \geq a \\ 0 & \text{if } h = 0 \end{cases} \tag{2.10}$$

The shape of this model is illustrated in Figure *2.7*.

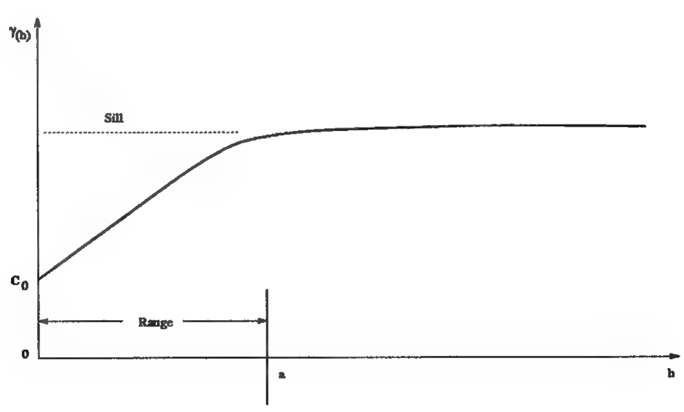

Figure 2.7 Spherical Model

Block Kriging. Block estimate is an estimate of the average value of a variable within the prescribed local area. So, "one method for finding such an estimate is to discretize the local area" (20:323). Since this method is simple but computationally time consuming, block kriging is developed as an alternative to this method by modifying the point kriging system.

Block kriging uses the same covariance matrix as point kriging. However, the covariance vector consists of covariance values between the random variables at the sample locations that we are trying to estimate. For block estimation, these covariances are point-to-block covariances while they are point-to-point covariances for point estimation. So, it is possible to convert a point kriging system to an ordinary block kriging system by making a single change (20:325). The main advantage of block kriging compared to point kriging is that it provides an estimate of the block average with the solution of only one kriging system; however, block kriging requires more computations than point kriging.

Universal Kriging. This technique is used when trend is present. There are three steps in Universal Kriging: 1) estimation and removal of the drift; 2) kriging of the nonstationary residuals to obtain needed estimates, and 3) combination of estimated residuals with the drift to obtain estimates of the actual surface (12:397).

Removing the drift can be done by estimating a polynomial for the drift and then subtracting this drift from the data. However, combining the neighborhood size with the drift equations makes this technique difficult to apply (16:259).

2.3.3 Conclusion.

This chapter provides a general review of the literature of linear programming and the basic aspects of techniques that are used by Johnson, *et al.* to create a methodology for performing optimality analysis. For optimality analysis, metamodels are developed by using experimental design and least squares regression. By

applying one of the kriging methods, these metamodels can be improved to get more accurate estimates.

III. Methodology

If the optimal objective function value is defined as the response, and the levels of RHS vectors are defined as predictors, RSM can be applied to predict the optimal objective function value based on the values of the elements of the RHS vector.

The response surface methodology uses experimental design and least squares regression to develop a simple polynomial model. Since this model is produced from another model, in our case from a large LP, it is called a metamodel (21:14). Metamodels are used to predict the optimal objective function value based on the values of the elements of the RHS vector since the optimal objective function value is defined as a function of the RHS vectors in the metamodels.

Figure 3.1 shows the metamodels for STORM that were developed in this thesis. The first metamodel describes the relationship between the objective function value, total flying cost, and the RHS vectors, tonnage requirement (TREQ) and frequency requirement (FREQ). The first factor, **TREQ**, is a 382×1 vector and the elements of the vector vary from 0.01 to 857 in tons. The second factor, **FREQ**, is a 151×1 vector whose elements range from 4.29 to 24.5 in visits per month between city pairs.

Table 3.1 Four Areas in the AMC Channel System

Area	Region
1	Central America, Hawaii, Guam, Caribbean, Alaska
2	Germany, England, Azores
3	Okinawa, Japan, South Korea, Italy, Spain
4	North Atlantic, Singapore, Australia, New Zealand, Saudi Arabia, Turkey, Israel

Metamodels 2, 3, and 4 show the relationships between the total cost and the tonnage and frequency requirements for four areas designated by AMC. Table *3.1* shows these areas. The **A1**, **A2**, **A3**, and **A4** vectors were simply derived from **TREQ** and **FREQ** RHS vectors via separating the elements of **TREQ** and **FREQ** vectors into the related areas. Thus, the sum of the number of elements in each area vector is equal to the number of elements in the **TREQ** and **FREQ** vectors.

Finally, metamodels 5 and 6 describe the relationship between the total flying cost and the region of interest in the dominating area associated with **TREQ** and **FREQ**. Region vectors were obtained from the area vector of interest in the same way as the area vectors obtained from the **TREQ** and **FREQ** vectors.

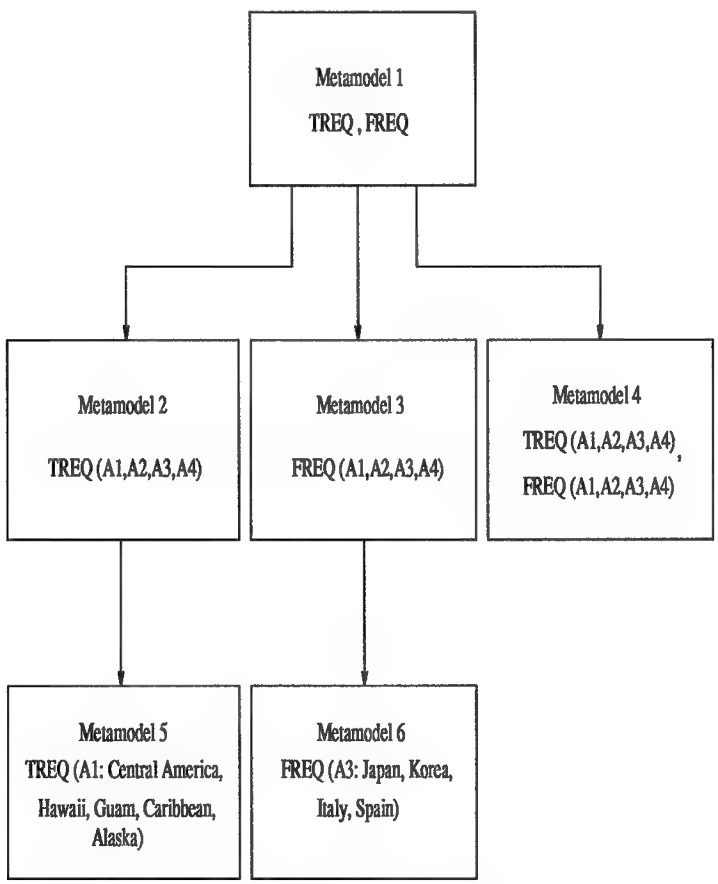

Figure 3.1   Research Directions

The following sections discuss how the techniques used in the methodology of Johnson, *et al.* as illustrated in Figure *3.2* are applied.

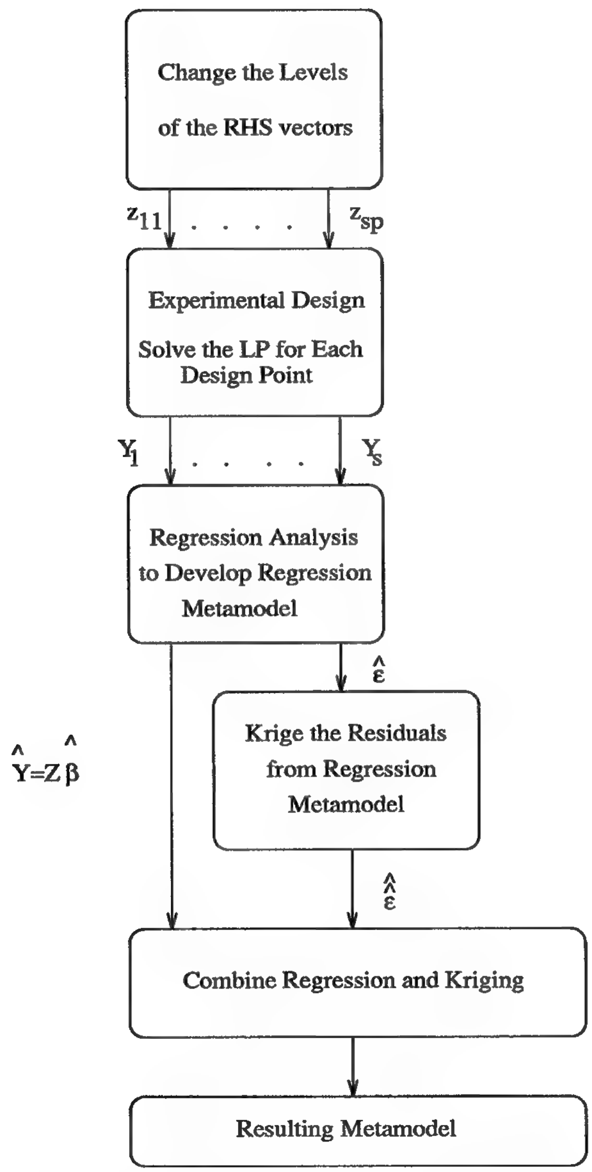

Figure 3.2 The Methodology of Johnson, *et al.* for Linear Programs

3.1 Experimental Design for Linear Programming

As stated in the second chapter, experimental designs are selected so that the experimental error variance is minimized. In our study, we obtain our response by solving STORM for each design point. In this case, no experimental error is associated with the solution from the LP model. Since there is no experimental error, the purpose is to minimize the misspecification bias in this research.

Minimization of the bias using minimum bias estimation was first suggested by Karson, Manson, and Hader (24). Karson, *et al.* uses an estimator which achieves minimum integrated squared bias. This estimator does not depend on the unknown coefficients, and it attains the same minimum integrated squared bias whatever the design is. When the "all bias" designs are used in experimental design, the minimum bias estimation of Karson, *et al.* is equal to the least squares estimation (6).

In this research, two experimental designs are used. The first design develops a data set containing the RHS vectors for the LP and the corresponding optimal objective function values obtained by solving the LP for each design point. This data set is called the test set. Metamodels are developed by applying least squares regression to the test set. The second experimental design is used to obtain a data set which is called the validation set. The validation set is chosen from the interior of the test set. Metamodels are validated by the second experimental design via comparing the optimal objective function values associated with the validation set to the predictions for the optimal objective function values obtained from metamodels.

The first step in the experimental design is to identify the admissible set of RHS vectors. Since the metamodels will only be valid over this admissible set of RHS vectors, it must be defined such that all RHS vectors of potential interest are included. In this study, admissible sets are constructed by increasing or decreasing the RHS vectors of interest by a prespecified percentage (usually 10 percent).

Table 3.2 Experimental Designs Used to Develop Metamodels In This Research

Metamodel	Number of Factors	Factorial Design
1	2 TREQ, FREQ	2^2 full
2	4 TREQ (A1,A2,A3,A4)	2_{IV}^{4-1} fractional
3	4 FREQ (A1,A2,A3,A4)	2_{IV}^{4-1} fractional
4	8 TREQ(A1-A4),FREQ(A1-A4)	2_{IV}^{8-4} and 2_{IV}^{8-3} fractional
5	5 TREQ(A1:CAm,Ha,Gu,Ca,Al)	2_{III}^{5-2} fractional
6	6 FREQ(A3:Ja,Ks,It,Sp)	2_{IV}^{4-1} fractional

The next step in the experimental design is to code the level of the constraints. Coding is achieved using a simple transformation given by

$$z_i = \frac{b_i - b_{i,0}}{S_i} \tag{3.1}$$

where $\mathbf{b_i}$ is the actual numerical level of the ith constraint. In this study, since we use two-level factorial designs, we can say that $\mathbf{b_i}$ has $\mathbf{b_{i,max}}$ and $\mathbf{b_{i,min}}$ indicating the higher and the lower levels of the constraints. Here $\mathbf{b_{i,0}}$ is the midpoint between $\mathbf{b_{i,max}}$ and $\mathbf{b_{i,min}}$. If $\mathbf{S_i}$ is taken to be $(\mathbf{b_{i,max}} - \mathbf{b_{i,min}})/2$, then $\mathbf{b_{i,max}}$ and $\mathbf{b_{i,min}}$ are mapped to 1 and -1, respectively.

The experimental designs used to develop the metamodels of interest in this research were determined according to the number of RHS vectors they were dealing with. These designs were derived from the tables by Box and Draper (5:164-165).

For example, to create a model which represents the relationship between four areas and the total flying cost due to tonnage requirement or frequency requirement, a 2_{IV}^{4-1} design was used since there are four factors associated with **TREQ** or **FREQ**. The designs used in this research are shown in Table *3.2*. The construction of these designs is described in the next chapter.

3.1.1 Least Squares Regression. In this research, we approximate the LP model, specifically STORM, with a simple model. To achieve this, we apply least squares regression to the data obtained from the experimental design phase of the methodology of Johnson, *et al.*Least squares regression develops an initial metamodel approximating $\mathbf{\Phi}$ with a first order or second order polynomial since those types of polynomials are able to define some concave surfaces such as hyperplanes and dome-shaped surfaces (21:19).

Generally, we wish to elucidate a model

$$\mathbf{y} = f(\boldsymbol{\xi}, \boldsymbol{\theta}) + \boldsymbol{\epsilon} \tag{3.2}$$

where

$$\mathbf{E}(\mathbf{y}) = \hat{\mathbf{y}} = f(\hat{\boldsymbol{\xi}}, \boldsymbol{\theta}) \tag{3.3}$$

is the mean level of the response $\mathbf{y}$ which is affected by k variables $(\xi_1, \xi_2, .., \xi_k) = \boldsymbol{\xi}^k$. The model also consists of q parameters $(\theta_1, \theta_2, .., \theta_q) = \boldsymbol{\theta}'$ and ϵ which is an experimental error due to the misspecification. To examine this model, first we make a series of experimental runs with s different sets of conditions, $\boldsymbol{\xi_1}, \boldsymbol{\xi_2}, .., \boldsymbol{\xi_s}$. Thus, we choose the design points, and then, we solve the LP model for each design point to get the response values of $y_1, y_2, .., y_s$.

If we compute $f(\xi, \theta)$ for each of the experimental runs $\boldsymbol{\xi_1}, \boldsymbol{\xi_2}, .., \boldsymbol{\xi_s}$, then we obtain s discrepancies $\{y_1 - f(\boldsymbol{\xi_1}, \theta)\}, \{y_2 - f(\boldsymbol{\xi_2}, \theta)\}, .., \{y_s - f(\boldsymbol{\xi_s}, \theta)\}$. Least squares regression selects the value that makes the sum of squares of these discrepancies as small as possible. Thus, it tries to find the best estimate of $\boldsymbol{\theta}$ in the form of

$$S(\boldsymbol{\theta}) = \sum_{u=1}^{s} \{y_u - f(\xi_u, \boldsymbol{\theta})\}^2 \tag{3.4}$$

where S($\boldsymbol{\theta}$) is called the sum of squares function. Note that, for any specific selection of one of the q parameters in $\boldsymbol{\theta}$, there is a specific value of S($\boldsymbol{\theta}$). The minimizing

selection of $\boldsymbol{\theta}$ is called the least squares estimate, and it is denoted by $\hat{\boldsymbol{\theta}}$. In general, the goodness of the least squares estimates of $\boldsymbol{\theta}$ depends on the distribution of the errors (6:35). For that reason, as stated in Chapter 2, the least squares estimates are appropriate if it is assumed that the experimental errors $\epsilon_u = y_u - \hat{y}_u, u = 1, 2, .., s$ are statistically independent, with constant variance, and normally distributed.

The computation of least squares estimates is very simple when the response function is linear in the parameters, with the form

$$\hat{y} = f(\xi, \theta) = \theta_1 z_1 + \theta_2 z_2 + \theta_q z_q \tag{3.5}$$

where the zs are known constants. In the experimental design, zs are known functions of the experimental conditions(runs), $\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, .., \boldsymbol{\xi}_s$. Equivalently we can say that they are the coded forms $x_i = (\xi_i - \xi_{i,0})/S_i, i = 1, 2, .., k$, where $\xi_{i,0}$ and S_i are suitable location and scale factors, respectively. Adding the experimental error $\epsilon = y - \hat{y}$, we have the model

$$y = f(\xi, \theta) = \theta_1 z_1 + \theta_2 z_2 + ... + \theta_q z_q + \epsilon \tag{3.6}$$

Box and Draper call this the linear model since it is linear in the parameters.

If Equation (3.5) has been postulated and experimental conditions $(z_{1u}, z_{2u}, .., z_{qu})$, $u = 1, 2, .., s$ have been run, yielding $y_1, y_2, .., y_s$, have been obtained, then the model relates the observations to the known z_{iu}'s and the unknown θ_i's by s equations

$$y_1 = \theta_1 z_{11} + \theta_2 z_{21} + \ldots\ldots + \theta_q z_{q1} + \epsilon_1 \tag{3.7}$$

$$y_2 = \theta_1 z_{12} + \theta_2 z_{22} + \ldots\ldots + \theta_q z_{q2} + \epsilon_2 \tag{3.8}$$

$$\vdots$$

$$y_s = \theta_1 z_{1s} + \theta_2 z_{2s} + \ldots\ldots + \theta_q z_{qs} + \epsilon_s \tag{3.9}$$

These equations can be written in matrix form as

$$\mathbf{y} = \mathbf{Z}\boldsymbol{\theta} + \boldsymbol{\epsilon} \tag{3.10}$$

where $\mathbf{Y}$ is an $s \times 1$ vector of responses, $\mathbf{Z}$ is an $s \times q$ matrix of known constants, $\boldsymbol{\theta}$ is the $q \times 1$ vector of unknown coefficients, and $\boldsymbol{\epsilon}$ is an $s \times 1$ vector of experimental error.

As we can see, the experimental design determines the $\mathbf{Z}$ matrix. To estimate the q parameters of the model separately, we should employ an experimental design that provides a $s \times q$ matrix with full column rank with $\mathbf{Z}'\mathbf{Z}$ nonsingular, so the solution to equations (3.7) through (3.9) can be written

$$\hat{\boldsymbol{\theta}} = (\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{y}, \tag{3.11}$$

and it is called the least squares estimator.

3.1.2 Kriging of the Residuals. It is possible to improve the metamodel obtained from the least squares regression by kriging. In this study, simple point kriging is applied to the residuals that are obtained from the test metamodel. Thus,

in Equation (2.7), $\mathbf{Y}$ is replaced by $\hat{\boldsymbol{\epsilon}}$, and $\hat{\mathbf{Y}}_{\mathbf{p}}$ is replaced by $\hat{\boldsymbol{\epsilon}}_p$ where $\hat{\boldsymbol{\epsilon}}$ is a $q \times 1$ vector of estimated residuals from the regression metamodel, and $\hat{\boldsymbol{\epsilon}}_p$ is the estimate of the regression residual to be obtained by kriging. So, the equation for estimation of the error can be written as

$$\hat{\boldsymbol{\epsilon}}_p = \mathbf{W}'\hat{\boldsymbol{\epsilon}} \tag{3.12}$$

The distance matrix is constructed by calculating the Euclidean distance between each design point. The covariance structure of the points are derived by using one of the variogram models summarized in Section *3.2.* For example, the spherical model can be applied to the distance matrix. By treating the residuals from the test metamodel as known points, the kriged residuals are estimated by finding the weights from Equation (2.9), and then applying these weights to Equation (3.13).

Note that the performance of point kriging depends on the covariance structure of the residuals. The values of the parameters that are used in the selected variograms *i.e.* C_0, C, and a in the spherical model, must be chosen so that the error is minimum.

3.1.3 Combination of Regression and Kriging. The estimated residuals obtained by kriging are simply added to the regression metamodels to get more accurate metamodels. Thus, we decrease the error between the true objective function value obtained by solving the LP and the objective function value predicted by the metamodel.

3.1.4 Measuring the Performance of the Metamodels. The predictions obtained from the regression metamodel and from the metamodel developed by using kriging in conjunction with regression are compared to the true optimal objective function values of the validation design by using the measures MSE, MAPE, and PC.

Mean squared error (MSE) is a measure of accuracy computed by squaring the individual error for each item in the data set, and then, finding the average or mean value of the sum of those squares. The mean squared error gives greater weight to large errors than to small errors since the errors are squared before being summed. MSE is defined by

$$MSE = \frac{\sum_{i=1}^{s}(\hat{Y}_i - Y_i)^2}{s}, \tag{3.13}$$

and it is an unbiased estimator of σ^2 for the regression model.

Mean absolute percentage error (MAPE) is the mean or average of the sum of all of the percentage errors for a given data set taken without regard to sign. Thus, their absolute values are summed and the average computed. MAPE is given as follows:

$$MAPE = \frac{1}{s}\sum_{i=1}^{s}\left|\frac{\hat{Y}_i - Y_i}{Y_i}\right| \times 100, \tag{3.14}$$

Percentage of predictions closer to the true optimal solution (PC) is defined by

$$PC = \frac{1}{s} \times \text{Number of Predictions Closer} \times 100 \tag{3.15}$$

PC is used to compare the regression metamodel to the improved metamodel obtained by both regression and kriging.

IV. Results and Discussion

This chapter presents the numerical results and their interpretations. In this thesis, six metamodels of STORM are developed to check the applicability of the methodology of Johnson, *et al.*. The test design, validation design, and kriging results are given for the first order designs. Then, the results concerning second order designs and their interpretations follow. The test metamodels are developed by perturbing the RHS vectors by 10 percent ($\Delta = \pm 0.1$). Refer to Figure *3.1* as an overview device for the following sections. Note that the metamodels are given in the coded form of the factors, and denoted by $\mathbf{z_i}, i = 1, ..., s$ in the following tables.

4.1 Metamodel 1

Metamodel 1 shows the effects of the factors, Tonnage Requirement (**TREQ**) and Frequency Requirement (**FREQ**) on the optimal objective function value, Total Flying Cost (in dollars). Here, **TREQ** is a RHS vector that contains 382 elements. Thus, there are 382 city pairs involved in cargo delivery. **FREQ** is a RHS vector containing 151 elements. This means that 151 city pairs have specified frequency levels that must be attained during a period. Appendix B presents the data sets of TREQ and FREQ.

4.1.1 First Order Design of Metamodel 1. Table *4.1* shows the experimental design and the regression results of Metamodel 1. Notice that R^2 is very close to 1, and the same amount of residual error occurs at every possible condition. The metamodel is given in coded form. We see that the coefficient of **TREQ** is approximately twice that of **FREQ**. Thus, a 10% change in the tonnage requirement changes the total flying cost twice as much as a 10% change in the frequency requirement.

Table 4.1 Metamodel 1: Test Design and Regression Results

2^2 Full Factorial Design					
run	**TREQ**	**FREQ**	**Y**	$\hat{\mathbf{Y}}$	$\hat{\epsilon}$
1	-1	-1	34,802,682	34,895,327	-92,645
2	1	-1	40,343,910	40,251,265	92,645
3	-1	1	37,607,931	37,515,286	92,645
4	1	1	42,778,578	42,871,223	-92,645

Source	df	Sum of Squares	Mean Square	R^2
Model	2	3.555×10^{13}	1.777×10^{13}	0.9990
Error	1	34,332,385,564	34,332,385,564	
C Total	3	3.55846×10^{13}		

$$\hat{\mathbf{Y}} = 38,883,275 + 2,677,969\mathbf{z_1} + 1,309,979\mathbf{z_2}$$

Table 4.2 Metamodel 1: Validation Design and Kriging Results

2^2 Full Factorial Design						
run	**TREQ**	**FREQ**	**Y**	$\hat{\hat{\epsilon}}$	$\hat{\mathbf{Y}}_{\mathbf{reg}}$	$\hat{\hat{\mathbf{Y}}}$
1	-0.5	-0.5	36,773,800	-26,680	36,889,301	36,862,621
2	0.5	-0.5	39,495,708	26,680	39,567,270	39,593,950
3	-0.5	0.5	38,128,297	26,680	38,199,280	38,225,960
4	0.5	0.5	40,747,355	-26,680	40,877,249	40,850,569

Table 4.3 Regression and Kriging Performance Comparison of Metamodel 1

	MSE	**MAPE**	**PC**
Regression	1.009×10^{10}	0.2501	50%
Regression + Kriging	9.433×10^{9}	0.2499	50%

If we recall that the original objective function value of STORM was \$38.69M, by looking at the regression metamodel in Table *4.1*, we can say that the contribution of the tonnage requirement to the total flying cost is \$2.68M while it is \$1.3M for the frequency requirement. In other words, the tonnage requirement changes the total flying cost by 7% while the frequency requirement changes the optimal objective function value by 3.5%. Also, it is possible to say that our regression metamodel estimates the total flying cost with an error of ±\$92645 for the design points taken into account.

To obtain a better metamodel, needless to say, a metamodel with less residual error, we apply simple point kriging to the residuals obtained from the regression metamodel (Refer to Appendix C for the application of simple point kriging). Then, we add these kriged residual estimates ($\hat{\epsilon}$) to the estimated optimal objective function values $\hat{\mathbf{Y}}$ of the validation design that are estimated by substituting the validation design points into the regression metamodel. The resulting objective function value vector is denoted by $\hat{\hat{\mathbf{Y}}}$ in Table *4.2*. Metamodel 1 was validated by a 2^2 full factorial design where the factors were coded at the (+0.5, -0.5) level (Perturbed by 5%). Figure *4.1* shows the test and validation design points on a coordinate axis. Note that the point (0,0) is the center point for both designs.

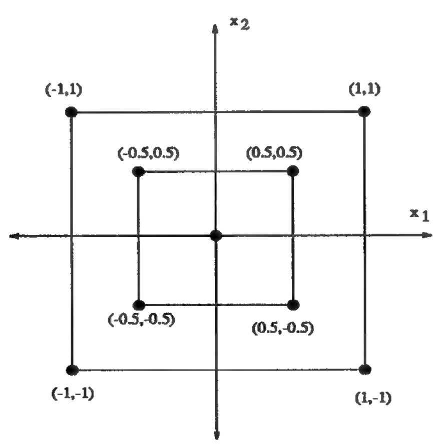

Figure 4.1 Test and Validation Designs of Metamodel 1

The comparison of the regression metamodel and the metamodel obtained by regression and kriging is given in Table *4.3*. When we compare $\hat{\mathbf{Y}}$ and $\hat{\hat{\mathbf{Y}}}$ to the true optimal objective function value vector, $\mathbf{Y}$, we see that $\hat{\hat{\mathbf{Y}}}$ of the first and the fourth design points are closer to $\mathbf{Y}$, while $\hat{\mathbf{Y}}$ of the second and the third design points are closer to $\mathbf{Y}$. Thus, the PC is 50% for both metamodels. Note that MAPE of the metamodels are very close to each other indicating that simple point kriging does not improve our regression metamodel much. The response surface is flat in the regression metamodel, and the metamodel explains our system with a small error so that we do not need to improve the metamodel furthermore. For that reason, kriging was not applied to the following metamodels.

4.1.2 Second Order Design of Metamodel 1. The second order ${2^{4-1}}_{IV}$ design of Metamodel 1 is given in Table *4.4*. The second order design with an interaction term fits the data without any type of error with one degree of freedom. The negative interaction term indicates that if we use the channelling cargo system effectively, it reduces the total flying cost by $92645 when the product of the tonnage and frequency requirements is one. When we examine STORM, we see that attaining frequency requirements without any cargo delivery causes an additional increase in the total flying cost. Thus, an effective channelling system rejects empty aircraft that have to be used to achieve the minimum required frequency level for a given period.

Table 4.4 Second Order Design of Metamodel 1

$$\hat{\mathbf{Y}} = 38,786,290 + 2,670,483\mathbf{z_1} + 1,303,072\mathbf{z_2} - 92,645\mathbf{z_1}\mathbf{z_2}$$

Regression	df	Type I Sum of Squares	R^2
Linear	2	3.556×10^{13}	0.9990
Quadratic	0	0	0.0000
Crossproduct	1	3.433×10^{10}	0.0010
Total Regression	3	3.558×10^{13}	1.0000

4.2 Metamodel 2

Metamodel 2 investigates the effects of the tonnage requirement of four areas on the total cost. These areas are abbreviated as **A1**, **A2**, **A3**, and **A4**, respectively, where **A1** is a 94×1 RHS vector, **A2** is a 39×1 RHS vector, **A3** is a 119×1 RHS vector, and finally, **A4** is a 130×1 RHS vector. Note that **TREQ** is composed of the four area vectors.

4.2.1 First Order Design of Metamodel 2. When we look at the Table *4.5*, we observe that R^2 is almost 1 showing that we have a good first order fit to the data. Also, the predicted optimal objective function values $\hat{\mathbf{Y}}$ are close to the true optimal objective function values $\mathbf{Y}$. The regression metamodel tells us that in case of a possible 10% change in tonnage requirement, Area 1 and Area 4 are the dominating factors for the total flying cost. Especially, A1 has the greatest effect on the total flying cost by 2.12% for the tonnage requirement. But, we should keep the dimensions of the area vectors in mind. For example, although **A2** seems the least important factor of all, it consists of only 39 city pairs. So, we can say that the those routes actually have great importance in cargo delivery.

4.2.2 Second Order Design of Metamodel 2. Table *4.6* summarizes the findings. We have a perfect fit of data as in Metamodel 1. Notice that Area 1 has an interaction with the other areas although the interaction effects are small. If we look at the map given in Appendix D which shows the channel cargo system of AMC, we see that Area 1 is generally the first leg for cargo movement to the the other areas. This causes the interaction of Area 1 with the other areas.

Table 4.5 Metamodel 2: First Order Design and Regression Results

$2^{4-1}{}_{IV}$ Fractional Factorial Design of **TREQ**							
run	**A1**	**A2**	**A3**	**A4**	**Y**	$\hat{\mathbf{Y}}$	$\hat{\epsilon}$
1	-1	-1	-1	-1	36,159,440	36,193,818	-34,378
2	1	-1	-1	1	39,495,836	39,479,478	16,358.4
3	-1	1	-1	1	38,748,515	38,753,033	-4,518.4
4	1	1	-1	-1	38,853,288	38,830,750	22,538.1
5	-1	-1	1	1	38,945,744	38,923,206	22,538.1
6	1	-1	1	-1	38,996,404	39,000,922	-4,518.4
7	-1	1	1	-1	38,290,836	38,274,478	16,358.4
8	1	1	1	1	41,525,760	41,560,138	-34,378

Source	df	Sum of Squares	Mean Square	R^2
Model	4	1.5159×10^{13}	3.7799×10^{12}	0.9997
Error	3	3,955,648,706	1,318,549,568.7	
C Total	7	1.5164×10^{13}		

$$\hat{\mathbf{Y}} = 38,876,978 + 840,844\mathbf{z_1} + 477,622\mathbf{z_2} + 562,708\mathbf{z_3} + 801,986\mathbf{z_4}$$

Table 4.6 Second Order Design of Metamodel 2

$$\hat{\mathbf{Y}} = 38,870,632 + 840,844\mathbf{z_1} + 475,868\mathbf{z_2} + 578,696\mathbf{z_3} + 787,752\mathbf{z_4} + 425.79\mathbf{z_1 z_2} - 19,448\mathbf{z_1 z_3} - 9,009.81\mathbf{z_1 z_4}$$

Regression	df	Type I Sum of Squares	R^2
Linear	4	1.511×10^{13}	0.9998
Quadratic	0	0	0.0000
Crossproduct	3	3.677×10^{9}	0.0002
Total Regression	7	3.5115×10^{13}	1.0000

Table 4.7 Metamodel 3: First Order Design and Regression Results

$2^{4-1}{}_{IV}$ Fractional Factorial Design of **FREQ**							
run	**A1**	**A2**	**A3**	**A4**	**Y**	$\hat{\mathbf{Y}}$	$\hat{\epsilon}$
1	-1	-1	-1	-1	37,434,501	37,542,443	-107,942
2	1	-1	-1	1	38,973,981	38,842,009	131,973
3	-1	1	-1	1	38,445,504	38,475,241	-29,737.5
4	1	1	-1	-1	38,461,651	38,455,945	5,706.7
5	-1	-1	1	1	39,241,418	39,235,712	5,706.7
6	1	-1	1	-1	39,186,678	39,216,415	-29,737.5
7	-1	1	1	-1	38,981,620	38,849,647	131,973
8	1	1	1	1	40,041,272	40,149,213	-107,942

Source	df	Sum of Squares	Mean Square	R^2
Model	4	3.9763×10^{12}	994,085,642,302	0.9851
Error	3	59,970,102,201	19,990,034,067	
C Total	7	4.0363×10^{12}		

$$\hat{\mathbf{Y}} = 38,845,828 + 320,067\mathbf{z_1} + 136,683\mathbf{z_2} + 516,919\mathbf{z_3} + 329,716\mathbf{z_4}$$

4.3 Metamodel 3

Metamodel 3 shows the key relationships among the roles of the four areas for the frequency requirement. Here, Area 1 is a 52×1 RHS vector, Area 2 is a 12×1 RHS vector, Area 3 is a 41×1 RHS vector, and Area 4 is a 46×1 RHS vector of **FREQ**. As in Metamodel 2, the **FREQ** data set is composed of these areas.

4.3.1 First Order Design of Metamodel 3. Table *4.7* shows the test design and regression results. Note that the R^2 is again close to 1.0 indicating that the metamodel fitted the data very well. For the frequency requirement, Area 3 which is composed of Japan, South Korea, Italy, and Spain has the greatest effect on the total flying cost by 1.3% when there is a 10% change in the frequency requirement.

4.3.2 Second Order Design of Metamodel 3. In Table *4.8*, we see that the second order metamodel fits the data perfectly again without any type of error with three degrees of freedom. Area 1 is the dominating factor for the frequency requirement as in the first order design of Metamodel 3. Notice that Area 1 has interactions with the other areas like the second order metamodel 2. However, in this case, Area 3 plays the most important role. Also, the interaction of Area 1 and Area 4 are significant since the frequency requirement is satisfied for Australia and New Zealand in Area 4 via Hawaii which is in Area 1.

Table 4.8 Second Order Design of Metamodel 3

$$\hat{\mathbf{Y}} = 38,970,828 + 445,067\mathbf{z_1} + 261,683\mathbf{z_2} + 641,919\mathbf{z_3} + 454,919\mathbf{z_4} + 73,883\mathbf{z_1}\mathbf{z_2} + 56,160\mathbf{z_1}\mathbf{z_3} - 137,015\mathbf{z_1}\mathbf{z_4}$$

Regression	df	Type I Sum of Squares	R^2
Linear	4	7.083×10^{12}	0.97
Quadratic	0	0	0.0000
Crossproduct	3	2.19×10^{11}	0.03
Total Regression	7	7.302×10^{12}	1.0000

4.4 Metamodel 4

This metamodel is the combination of Metamodel 1 and Metamodel 2. It yields the same insights as Metamodels 1 and 2 since we get the same results from the first order design. They are shown in Table *4.9.* Note that it is easy to see that Metamodels 1 and 2 were developed correctly.

Table 4.9 Metamodel 4: First Order Design and Regression Results

2^{8-4}_{IV} Fractional Factorial Design of **TREQ and FREQ A1 A2 A3 A4**											
run	**TA1**	**TA2**	**TA3**	**TA4**	**FA1**	**FA2**	**FA3**	**FA4**	**Y**	$\hat{\mathbf{Y}}$	$\hat{\epsilon}$
1	-1	-1	-1	-1	-1	-1	-1	-1	34802682	34996611	-193929
2	1	-1	-1	-1	-1	1	1	1	38584729	38616983	-32253.6
3	-1	1	-1	-1	1	-1	1	1	38326536	38199785	126751
4	1	1	-1	-1	1	1	-1	-1	38542247	38520104	22143.1
5	-1	-1	1	-1	1	1	1	-1	38219106	38125097	94009.1
6	1	-1	1	-1	1	-1	-1	1	39258659	39169893	88765.6
7	-1	1	1	-1	-1	1	-1	1	37986503	38035961	-49457.7
8	1	1	1	-1	-1	-1	1	-1	39679532	39735561	-56029
9	-1	-1	-1	1	1	1	-1	1	38177527	38233556	-56029
10	1	-1	-1	1	1	-1	1	-1	39883699	39933156	-49457.7
11	-1	1	-1	1	-1	1	1	-1	38887989	38799224	88765.6
12	1	1	-1	1	-1	-1	-1	1	39938030	39844021	94009.1
13	-1	-1	1	1	-1	-1	1	1	39471157	39449014	22143.1
14	1	-1	1	1	-1	1	-1	-1	39896084	39769332	126751
15	-1	1	1	1	1	-1	-1	-1	39319881	39352135	-32253.6
16	1	1	1	1	1	1	1	1	42778578	42972506	-193929

Source	df	Sum of Squares	Mean Square	R^2
Model	8	3.8215×10^{13}	4.7768×10^{12}	0.9960
Error	7	1550142525548	22144893221	
C Total	15	3.8369×10^{13}		

$$\hat{\mathbf{Y}} = 38,984,559 + 835,636\mathbf{z_1} + 447,853\mathbf{z_2} + 591,629\mathbf{z_3} + 809,559\mathbf{z_4} \\ +328,720\mathbf{z_5} + 149,537\mathbf{z_6} + 494,357\mathbf{z_7} + 330,656\mathbf{z_8}$$

4.5 Metamodel 5

The following two metamodels can be called the regional search metamodels since they are trying to find the key relationships between the regions and the total cost in the dominating areas, namely Area 1 for the tonnage requirement and Area 3 for the frequency requirement.

Metamodel 5 investigates the effect of Central America, Hawaii, Guam, Caribbeans, and Alaska on the total flying cost. In Figure *4.10*, we see that Central America is able to change the objective function value by 1%. Since there is a good fit to the data with $R^2 = 1.0$, we do not need to examine the second order metamodel although there is an insignificant interaction between Hawaii and the other regions because Hawaii is an important stopover point in the middle of the Pacific Ocean.

Table 4.10 Metamodel 5: First Order Design and Regression Results

2^{5-2}_{III} Fractional Factorial Design of **TREQ Area 1**								
run	**CAm**	**HA**	**GU**	**CAR**	**AL**	**Y**	$\hat{\mathbf{Y}}$	$\hat{\epsilon}$
1	-1	-1	-1	1	1	38,450,021	38,450,303	-281.3
2	1	-1	-1	-1	-1	38,732,340	38,736,397	-4,057.2
3	-1	1	-1	-1	1	38,360,306	38,360,025	281.3
4	1	1	-1	1	-1	39,309,170	39,305,112	4,057.2
5	-1	-1	1	1	-1	38,354,151	38,353,869	281.3
6	1	-1	1	-1	1	38,993,751	38,989,694	4,057.2
7	-1	1	1	-1	-1	38,263,310	38,263,592	-281.3
8	1	1	1	1	1	39,554,352	39,558,409	-4,057.2

Source	df	Sum of Squares	Mean Square	R^2
Model	5	1.6547×10^{12}	330937442922	1.0000
Error	2	66,160,077.761	33,080,038.881	
C Total	7	1.65475×10^{12}		

$$\hat{\mathbf{Y}} = 38,752,175 + 395,228\mathbf{z_1} + 119,609\mathbf{z_2} + 39,216\mathbf{z_3} + 164,748\mathbf{z_4} + 87,433\mathbf{z_5}$$

4.6 Metamodel 6

This metamodel was developed to observe the regional effects on **FREQ** of Area 3 since this area is dominant on the total flying cost as we observe from Metamodel 3.

4.6.1 First Order Design of Metamodel 6.

Tables 4.11 and 4.12 show that Japan and Italy are the dominating regions on the total cost by 0.65% for a 10% change in the frequency requirements. The map in Appendix D showing the AMC's Cargo Channel System confirms our statement.

Table 4.11 Metamodel 6: First Order Design and Regression Results

$2^{4-1}{}_{IV}$ Fractional Factorial Design of **FREQ Area 3**							
run	**JAPAN**	**SKOREA**	**ITALY**	**SPAIN**	**Y**	$\hat{\mathbf{Y}}$	$\hat{\epsilon}$
1	-1	-1	-1	-1	38,213,241	38,229,721	-16,479.9
2	1	-1	-1	1	38,760,857	38,744,381	16,475.8
3	-1	1	-1	1	38,284,713	38,268,245	16,467.7
4	1	1	-1	-1	38,766,451	38,782,914	-16,463.5
5	-1	-1	1	1	38,700,791	38,717,254	-16,463.5
6	1	-1	1	-1	39,248,391	39,231,923	16,467.7
7	-1	1	1	-1	38,772,263	38,755,787	16,475.8
8	1	1	1	1	39,253,968	39,270,448	-16,479.9

Source	df	Sum of Squares	Mean Square	R^2
Model	4	1.0081×10^{12}	252028629857	0.9979
Error	3	2,170,540,199.8	723,513,399.93	
C Total	7	1.0103×10^{12}		

$$\hat{\mathbf{Y}} = 38,750,084 + 257,332\mathbf{z_1} + 19,264\mathbf{z_2} + 243,769\mathbf{z_3} + 0\mathbf{z_4}$$

4.6.2 Second Order Design of Metamodel 6. In Figure *4.12*, we see that the interaction between Japan and South Korea is significant because these channels serve the same region of interest.

Table 4.12 Second Order Design of Metamodel 6

$\hat{\mathbf{Y}} = 38750084 + 257332\mathbf{z_1} + 19264\mathbf{z_2} + 243769\mathbf{z_3} + 0\mathbf{z_4} - 16472\mathbf{z_1}\mathbf{z_2}$			
Regression	df	Type I Sum of Squares	R^2
Linear	4	1.008×10^{12}	0.9979
Quadratic	0	0	0.0000
Crossproduct	3	2.17×10^{9}	0.0021
Total Regression	7	1.01×10^{12}	1.0000

V. *Conclusions and Recommendations*

5.1 Conclusions

The methodology developed by Johnson, Bauer, Moore, and Grant for the optimality analysis of LPs works very well for the large scale linear programming models as was shown in Chapter 4. The metamodels developed for different scenarios provided some useful insights about the LP model, namely STORM, and the metamodels described the key relationships between the optimal objective function value, the total flying cost, and the right-hand-side vectors of interest.

The simple point kriging did not contribute a considerable amount of improvement to the developed metamodels not because it did not work in general, but because of the structure of the STORM model. Note that the response surface of the optimal objective function value is a flat surface. For that reason, the optimal objective function value can be estimated by a simple polynomial with remarkable accuracy as we have seen in our study.

Although the metamodels we have developed may suggest a change in the optimal routing system to minimize the total flying cost when there is a 10% change in the tonnage and frequency requirements, we saw that the gain from changing the resources by 10% or from making some changes in the routing system is insignificant if we compare the results with the total flying cost which is almost $40M.

5.2 Recommendations

5.2.1 Recommendations for AMC. This thesis effort limited itself by changing the RHS vectors of STORM by 10%. One may study perturbing the RHS vectors of interest for various levels of change. To give an incentive, the effect of the frequency requirement (**FREQ**) on the total flying cost is summarized in Table *5.1* when the change in **TREQ** is kept at constant 10% level while the **FREQ** is changed by 50%,

Table 5.1 Effect of **TREQ** on the Total Cost for Various Perturbations

Effect on the Optimal Objective Function Value		
Δ	In Million Dollars	In Percentage
± 0.50	6.47	16.7%
± 0.80	10.72	27.7%
± 1.00	14.36	37.1%

80%, and 100% (100% change means the values of **TREQ** are doubled or set to zero).

5.2.2 Recommendations About the Application of the Methodology.

1. Excel 5.0 was used to change the levels of the RHS data set. Although it was very efficient, one may use the Macro utilities of the related software to make the data manipulations in a shorter time.
2. Simple point kriging was applied to the regression metamodel residuals in this study. Other types of kriging techniques presented in Chapter 2, for example universal kriging, can be used in further studies.
3. During the application of kriging, other types of models can be used rather than the spherical model to get better estimates.

5.3 Further Study

The methodology of Johnson, *et al.* can also be applied to the objective function coefficients of LPs since the objective function values are simply the duals of the RHS vector values in a linear programming model.

Appendix A. STORM

This information about STORM was provided by Mr. Alan Whisman of the Command Analysis Group at AMC.

The Data Requirements for the STORM Model

1. Aircraft Routes and Flying Times: Possible cargo routes must be specified to the STORM model in advance, along with flying times for each type of plane on each route.

2. Aircraft Data: Information specific to each aircraft type, such as payload, total number of flying hours available, and cost per operating hour. Aircraft include commercial cargo planes (Boeing 747, DC-8, and DC-10) as well as AMC military aircraft (C-5, C-141, and C-130).

3. Cargo and Frequency Requirements: The number of missions to be flown is based upon the need to move aircraft and cargo between certain pairs of bases. Cargo tonnage, frequency requirements, and possible transshipment points must be specified for each pair of bases.

4. Cargo Handling Costs: The STORM model allows cargo to move either by a direct flight or utilizing one transshipment. Each time cargo is loaded or unloaded, a handling cost of $176 per ton is incurred.

5. Base Operating Limits: If any base in the system has restrictions on the maximum number of channel flights it can handle in a month, this value must be specified.

The STORM model uses the preceding information to solve a linear programming problem which can best be described as follows:

MINIMIZE: Total costs incurred for aircraft operating hours and cargo handling.

SUBJECT TO:

1. Meeting as many of the cargo movement requirements as possible.
2. Meeting all requirements for service frequency.
3. Operating within each aircraft type's flying hour and payload limits.
4. Operating within each base's limit on monthly sorties.

Outputs of the model include:

1. The number of missions flown on each route by each aircraft type.
2. The number of tons that could not be delivered for each origin/destination pair. Since STORM allows only one transshipment for each piece of cargo, many of these undelivered requirements can be met with two transshipments.
3. For each movement requirement, the tonnage delivered directly by a single mission.
4. For each movement requirement, the tonnage transshipped through each possible transshipment point.
5. The percentage of available cargo space being utilized on each leg of each route.

The linear programming formulation used in STORM has been implemented in the LP modeling language GAMS. GAMS is available for a wide variety of LP solvers and machines, and greatly decreases the time involved in formulating, testing, and debugging models, as well as providing an excellent report writing capability. The description given below is designed to aid in interpreting the GAMS program listing.

There are six types of variables that are used in the STORM LP model of the channel cargo system. They are as follow:

1. Y(c,cd): The number of tons of cargo from source base 'c' which cannot be delivered to final destination bases 'cd'.

2. X(routes, aircraft): The number of missions to be flown on a route by aircraft type.

3. D(routes, stp, st2): The number of tons of cargo to be delivered directly from stop number 'stp' of the route (its origin) to stop number 'st2' of the route (its destination).

4. S(routes, stp, st2): The number of tons picked up at a transshipment point (stop number 'stp' of the route) and dropped at its final destination (stop number 'st2').

5. T(routes, stp, st2, cd): The number of tons picked up at its origin (stop 'stp') and dropped off at a transshipment point (stop 'st2') for eventual delivery to its final destination base 'cd'.

6. SLK(routes, stp): The unused cargo capacity on the 'stp'th leg of the route.

Generating variables for all combinations of routes, bases and aircraft types would yield an unmanageable number of redundant and unnecessary variables in the LP formulation. To prevent this, a FORTRAN program examines the set of routes, transshipment points, and cargo requirements and builds only those combinations required for the current data. The FORTRAN program builds a number of data sets which are used as "INCLUDE" input files in the GAMS program. The created files have extensions .TMP and .VAR.

There are nine types of constraints in the model. They are listed below by their GAMS names and described briefly:

1. TREQ(c, cd): Total tonnage to be moved from base 'c' to base 'cd' must equal the direct deliveries plus the transshipment deliveries plus the amount left undelivered.

2. CARGB(ct, cd): The amount of cargo being unloaded at base 'ct' for transshipment to base 'cd' must equal the amount being picked up at 'ct' for transshipment delivery to base 'cd'.

3. PL1(routes): Cargo carried on the first leg of a route cannot exceed the sum of the payloads of all aircraft flying that route.
4. PLP(routes, stp): Cargo loaded to fly leg 'stp' ($stp > 1$) cannot exceed the unused capacity on the previous leg plus the amount of cargo just unloaded.
5. ABAL(c, aircraft): At each base 'c' the number of landings by an aircraft type must equal the number of takeoffs by that type (conservation of flow).
6. FREQ(c, cd): The required minimum number of direct flights (i.e. the frequency requirement) from base 'c' to base 'cd' must be attained.
7. UPHRS(aircraft): Maximum number of flying hours for each aircraft type must not be exceeded.
8. LOHRS(aircraft): Minimum number of flying hours for each aircraft type must be achieved.
9. MAXLND(c): The maximum allowable visits to a base 'c' must not be exceeded.

The objective function, COST, is the sum of (a) aircraft operating hours, (b) a surcharge of 79% on flying hours for commercial missions that are bought one-way, (c) a handling charge for each time a ton of cargo is loaded or unloaded, and a "penalty cost" of $10,000 per ton for failure to deliver cargo.

STORM Model in GAMS

```
* STORM MODEL
$OFFSYMXREF OFFSYMLIST
OPTIONS SOLPRINT=OFF, SYSOUT=OFF,RESLIM=10000 ;
SETS
  aircraft PLANE TYPES /
$INCLUDE "planes.stm"
  /
```

```
  stp NUMBERED SEQUENCE OF STOPS IN ROUTE /01 * 18/ ;
ALIAS (stp,st2);
SET routes AIRCRAFT ROUTES /
$INCLUDE "route.tmp"
  / ;
SET c CITIES /
$INCLUDE "bases.stm"
   / ;
ALIAS (c,cd,ct) ;
SETS tp(ct,cd) TRANSSHIPMENT BALANCE PAIRS /
$INCLUDE "trans.tmp"
   /
    dvar(routes,stp,st2) COMBINATIONS FOR DIRECT DELIVERY /
$INCLUDE "d.var"
   /
    svar(routes,stp,st2) SECOND HALF OF TRANSSHIPMENT DELIVERY /
$INCLUDE "s.var"
   /
    tvar(routes,stp,st2,cd) FIRST HALF OF TRANSSHIPMENT DELIVERY /
$INCLUDE "t.var"
   /
    onrte(routes,stp,st2,c,ct) EACH ROUTES SEQUENCE OF STOPS /
$INCLUDE "onroute.tmp"
   /
    start(routes,c) STARTING CITY OF EACH ROUTE /
$INCLUDE "start.tmp"
   /
    ends(routes,c) ENDING CITY OF EACH ROUTE /
```

```
$INCLUDE "ends.tmp"
   / ;

PARAMETERS LENGTH(routes) NUMBER OF CITIES VISITED BY ROUTE /
$INCLUDE "lengths.tmp"
   /
           V(routes,c) NUMBER OF VISITS TO EACH CITY BY EACH ROUTE /
$INCLUDE "visits.tmp"
   /
           TC(c,cd) TOTAL CARGO TO BE MOVED BETWEEN CITIES /
$INCLUDE "load.tmp"
   /
           CF(c,cd) FREQUENCY REQS BETWEEN CITIES /
$INCLUDE "freq.tmp"
   / ;

TABLE H(routes,aircraft) FLYING HOUR ARRAY
$INCLUDE "fy94aug.tms"
     ;

PARAMETER ML(c) MAXIMUM NUMBER OF MISSIONS TO EACH CITY ;
ML(c) =1000;

TABLE AC(aircraft,*) AIRCRAFT INFORMATION

$INCLUDE "misc.stm"
     ;
```

```
SCALAR B  CARGO HANDLING COST PER TON /176.0/ ;
SCALARS CARGUNM, CARGTOT ;
 VARIABLE Z ;
POSITIVE VARIABLES
    X(routes,aircraft) SORTIES ON A ROUTE BY AN AIRCRAFT TYPE
    Y(c,cd) UNDELIVERED CARGO
    D(routes,stp,st2) DIRECT DELIVERY CARGO
    T(routes,stp,st2,cd) FIRST PART OF TRANSSHIPMENT DELIVERY
    S(routes,stp,st2) SECOND PART OF TRANSSHIPMENT DELIVERY
    SLK(routes,stp) UNUSED CAPACITY ON EACH LEG OF EACH ROUTE ;

*** BOUNDS ON 'X' VARIABLES INCLUDED IN FILE 'LXODA.STM'.
$INCLUDE "lx.stm"
EQUATIONS
  TREQ(c,cd) tonnage required to move between city pairs
  CARGB(ct,cd) cargo transshipped in equals cargo transhipped out
  PL1(routes) capacity constraint on first leg of a route
  PLP(routes,stp) capacity constraint on other legs of route
  ABAL(c,aircraft) landings equals takeoffs for each plane type
  FREQ(c,cd) meet frequency requirements
  UPHRS(aircraft) upper limit on aircraft flying hours
  LOHRS(aircraft) lower limit on aircraft flying hours
  MAXLND(c) maximum number of landings at any base
  COSTS objective function of flying and handling costs ;

TREQ(c,cd)$TC(c,cd).. Y(c,cd) +
   SUM((routes,stp,st2)$(dvar(routes,stp,st2)$onrte(routes,stp,st2,c,cd)),
       D(routes,stp,st2)) +
```

```
   SUM((routes,stp,st2,ct)$(tvar(routes,stp,st2,cd)$onrte(routes,stp,st2,c,ct)),
       T(routes,stp,st2,cd)) =G= TC(c,cd) ;


CARGB(ct,cd) $tp(ct,cd)..
   SUM((routes,stp,st2,c) $(tvar(routes,stp,st2,cd)$onrte(routes,stp,st2,c,ct)),
       T(routes,stp,st2,cd)) =E=
   SUM((routes,stp,st2) $(svar(routes,stp,st2)$onrte(routes,stp,st2,ct,cd)),
       S(routes,stp,st2)) ;


PL1(routes).. SUM(aircraft $H(routes,aircraft),
       AC(aircraft,"PAY") * X(routes,aircraft)) =E= SLK(routes,"01") +
  SUM(st2 $dvar(routes,"01",st2), D(routes,"01",st2)) +
  SUM(st2 $svar(routes,"01",st2), S(routes,"01",st2)) +
  SUM((st2,cd) $tvar(routes,"01",st2,cd), T(routes,"01",st2,cd)) ;


PLP(routes,stp)$(ORD(stp) lt LENGTH(routes) and ORD(stp) ge 2)..
          SLK(routes,stp-1) +
  SUM(st2 $dvar(routes,st2,stp), D(routes,st2,stp)) +
  SUM(st2 $svar(routes,st2,stp), S(routes,st2,stp)) +
  SUM((st2,cd) $tvar(routes,st2,stp,cd), T(routes,st2,stp,cd))


      =E= SLK(routes,stp) +
  SUM(st2 $dvar(routes,stp,st2), D(routes,stp,st2)) +
  SUM(st2 $svar(routes,stp,st2), S(routes,stp,st2)) +
  SUM((st2,cd) $tvar(routes,stp,st2,cd), T(routes,stp,st2,cd)) ;


ABAL(c,aircraft)..
   SUM(routes$(start(routes,c) $H(routes,aircraft)),X(routes,aircraft))
```

```
                        =E=
   SUM(routes$(ends(routes,c) $H(routes,aircraft)),X(routes,aircraft)) ;

FREQ(c,cd) $CF(c,cd)..
   SUM((routes,aircraft)$H(routes,aircraft),
       SUM((stp,st2)$onrte(routes,stp,st2,c,cd),
          X(routes,aircraft))) =G= CF(c,cd) ;

UPHRS(aircraft).. SUM(routes $H(routes,aircraft),
      H(routes,aircraft) * X(routes,aircraft))
        =L= AC(aircraft,"FH") ;

LOHRS(aircraft).. SUM(routes $H(routes,aircraft),
      H(routes,aircraft) * X(routes,aircraft))
        =G= AC(aircraft,"LH") ;

MAXLND(c)$(ML(c) le 1000).. SUM((routes,aircraft)
    $H(routes,aircraft), V(routes,c) * X(routes,aircraft)) =L= ML(c) ;

COSTS.. Z =E= SUM((routes,aircraft) $H(routes,aircraft),
       AC(aircraft,"COST") * H(routes,aircraft) * X(routes,aircraft))
  + SUM((routes,aircraft) $(start(routes,"EXXX") $H(routes,aircraft)),
    0.79 * AC(aircraft,"COST") * H(routes,aircraft) * X(routes,aircraft))
    + B * SUM((routes,stp,st2)$dvar(routes,stp,st2),D(routes,stp,st2))
    + 2 * B * SUM((routes,stp,st2)$svar(routes,stp,st2),S(routes,stp,st2))
    + 20000 * SUM((c,cd)$TC(c,cd),Y(c,cd)) ;

MODEL STORM1 /ALL/ ;
```

```
OPTION ITERLIM = 40000 ;

OPTION LP = ZOOM;

SOLVE STORM1 USING LP MINIMIZING Z ;

DISPLAY Z.L ;
```

TREQ Data File

KDOV.LETO	22.12
KDOV.LIPA	173.77
KSUU.FJDG	40.33
LETO.KDOV	58.26
EDAR.KWRI	9.60
EDAR.KNGU	23.60
EDAR.KDOV	947.49
EDAF.KDOV	923.74
KDOV.EDAR	744.00
KDOV.EDAF	787.57
KDOV.LTAG	140.19
RJSM.KSUU	57.58
KSUU.RJSM	108.69
PHIK.KSUU	451.39
KSUU.PHIK	545.56
KDOV.OERY	199.12
KDOV.LGIR	9.07
KSUU.RKJK	32.71
KSUU.RKPK	20.47
EGUN.KCHS	345.78
OEDR.KDOV	106.79
KDOV.OEDR	323.47
PKWA.KSUU	27.65
KSUU.RODN	392.99
KSUU.RKSO	634.20

KSUU.PGUA	196.14
KSUU.RJTY	340.32
RODN.KSUU	254.33
RJTY.KSUU	338.31
KSUU.PAED	239.52
KSUU.RJOI	25.02
KSUU.PJON	11.91
KTCM.PAED	62.50
KTCM.PADK	81.12
PADK.KTCM	9.96
RJTY.PHIK	13.83
RJTY.PGUA	25.51
RJTY.WSAP	28.43
KSUU.WSAP	18.68
WSAP.KSUU	6.92
RJTY.FJDG	162.67
RJTY.RKJK	0.49
RJTY.RKSO	50.90
RJTY.RJFF	13.58
RJTY.RJOI	27.51
RJTY.RJAW	8.71
RJTY.RODN	69.03
RJTY.RJSM	159.37
RJTY.RJCB	1.66
RJTY.RJAM	2.68
PAED.KSUU	330.32
PHIK.PGUA	17.56
PHIK.RODN	17.69

PHIK.PJON	163.57
PHIK.PKWA	122.46
PHIK.PWAK	70.38
PHIK.PMDY	11.82
PGUA.KSUU	83.91
NZCH.KSUU	20.14
FJDG.KSUU	95.26
FJDG.RJTY	27.14
FJDG.OMFJ	15.76
OMFJ.FJDG	0.01
FJDG.WSAP	17.10
WSAP.FJDG	12.69
WSAP.RJTY	2.31
RKSO.KSUU	647.15
RKSO.RJTY	28.92
RKSO.RODN	25.70
RJFF.RJTY	0.07
RJOI.KSUU	14.30
RJOI.RODN	26.18
RJAW.RJTY	11.86
RODN.RJTY	87.59
RODN.PHIK	25.37
RODN.RKPK	2.91
RODN.RKJK	3.69
RODN.RKSO	41.46
RODN.RJOI	30.77
RODN.RJSM	3.68
RODN.PGUA	20.17

PGUA.RODN	9.89
RJSM.RJTY	52.20
RJSM.RKJK	0.03
RJSM.RODN	2.10
RJCB.RJTY	1.77
ABAS.KSUU	18.58
APLM.ASRI	0.01
APLM.KSUU	31.68
ASRI.APLM	2.57
ASRI.KSUU	50.51
APWR.KSUU	17.24
RJAM.RJTY	1.00
KSUU.NZCH	22.55
KSUU.ABAS	106.86
KSUU.ASRI	34.67
KSUU.APWR	11.04
PJON.PHIK	30.43
PKWA.PHIK	73.30
PWAK.PHIK	0.39
PMDY.PHIK	4.49
KSUU.VTBD	9.09
RODN.VVNB	0.01
VVNB.RODN	0.01
RODN.VTBD	17.53
KCHS.SBGL	1.59
SBGL.KCHS	6.94
KCHS.SEQU	14.81
RJFF.KSUU	6.41

KSUU.RJFF	16.32
KDOV.RODN	8.87
KCHS.SVMI	9.31
SKBO.KCHS	12.01
KSUU.WIIH	5.06
WIIH.KSUU	4.33
KCHS.MHTG	17.25
MHTG.KCHS	6.68
KCHS.EGUN	292.91
KCHS.MHSC	216.94
KCHS.MPHO	493.74
KCHS.MSSS	52.38
KDOV.OJAF	8.64
KDOV.HECA	55.52
KNGU.LERT	132.13
KNGU.LIRN	64.63
KNGU.LICZ	105.19
KNGU.OBBI	63.69
KNGU.BIKF	142.76
KNGU.MUGM	271.30
KNGU.TJNR	211.93
KNGU.TXKF	128.44
KNGU.OMFJ	40.88
OMFJ.KNGU	0.01
KWRI.LPLA	285.47
KWRI.BGTL	146.24
EDAR.EGUN	58.06
EDAR.LETO	7.40

EDAR.LTAG	17.96
EDAR.HECA	21.67
EDAR.LIPA	51.65
EDAF.EGUN	81.32
EDAF.LETO	5.97
EDAF.OEDR	162.35
EDAF.LTAG	211.83
EGUN.EDAR	27.97
EGUN.LETO	3.74
EGUN.LIPA	4.84
EGUN.LTAG	18.42
LERT.KNGU	138.37
LERT.LICZ	12.32
LETO.EDAR	2.31
LETO.EDAF	10.02
LETO.EGUN	0.88
LETO.LIPA	16.57
LPLA.KWRI	99.07
LIPA.EDAR	0.05
LIPA.EGUN	0.55
LIPA.LETO	0.02
LIRN.KNGU	0.01
LIRN.LIEO	0.01
KNGU.LIPA	21.26
LIRN.LGSA	0.01
LGSA.LIRN	0.01
LIEO.LIRN	0.01
LIRN.LICZ	0.01

LICZ.KNGU	143.87
LICZ.LIRN	18.47
LTAG.KDOV	176.99
LTAG.EDAR	101.43
LTAG.EDAF	60.28
LTAG.EGUN	71.51
OJAF.KDOV	5.19
OBBI.KNGU	110.03
OEDR.EDAF	15.09
BGTL.KWRI	156.18
MHSC.KCHS	83.21
MHSC.MPHO	29.94
MPHO.KCHS	356.59
MPHO.MHSC	109.61
MUGM.KNGU	34.81
TJNR.KNGU	42.64
TJNR.MTPP	0.01
TJNR.TAPA	1.07
TAPA.TJNR	0.01
TJNR.TISX	0.01
TISX.TJNR	0.01
TXKF.KNGU	28.11
KWRI.CYYT	0.01
CYYT.KWRI	0.01
KCOF.FHAW	77.06
KCOF.TAPA	52.91
TAPA.KCOF	6.53
KNGU.HKNA	3.70

PAED.PADK	10.15
PHIK.ASRI	13.12
KCHS.SLLP	37.02
MPHO.SGAS	1.40
SGAS.MPHO	0.16
MPHO.SKBO	9.42
SKBO.MPHO	0.35
MPHO.SAEZ	4.59
SAEZ.MPHO	0.04
MPHO.MGGT	8.39
MGGT.MPHO	0.02
MPHO.SPIM	12.90
SPIM.MPHO	0.60
MPHO.SLLP	14.02
SLLP.MPHO	0.89
MPHO.MNMG	14.06
MPHO.SVMI	0.71
MPHO.SUMU	0.03
MPHO.MROC	3.20
MPHO.SBGL	0.74
MPHO.MSSS	23.18
MSSS.MPHO	0.01
MPHO.SCEL	0.65
SCEL.MPHO	0.01
MPHO.MHTG	19.69
MHTG.MPHO	0.13
MPHO.SEQU	12.63
SEQU.MPHO	0.50

RJTY.PAED	16.60
MPHO.SBBR	6.22
PWAK.KSUU	6.53
RKJK.KSUU	25.67
HECA.KDOV	16.39
HECA.EDAR	5.62
KCHS.SGAS	1.17
KCHS.SAEZ	0.10
SAEZ.KCHS	0.01
KCHS.GOOY	10.64
KCHS.FZAA	2.55
SLLP.KCHS	0.39
KCHS.SUMU	0.32
SUMU.KCHS	0.01
KCHS.FTTJ	4.24
KCHS.SCEL	9.18
SCEL.KCHS	11.92
LIPA.KDOV	3.41
KDOV.LLBG	5.65
PAED.RJTY	18.74
MROC.MPHO	3.00
SBGL.MPHO	7.01
RKSO.RKJK	10.88
EDAR.OJAF	19.14
EDAR.OEDR	14.08
KSUU.NSTU	1.10
PJON.KSUU	18.63
KSUU.PKWA	70.41

PAED.KTCM	14.41
MNMG.MPHO	0.01
SVMI.MPHO	0.23
SUMU.MPHO	0.09
KCHS.GLRB	0.34
GLRB.KCHS	0.01
KCHS.MUGM	14.82
KCHS.DRRN	5.53
KCHS.TJNR	8.68
KCHS.SPIM	14.93
SPIM.KCHS	3.67
KDOV.EGUN	6.08
KTCM.RODN	10.71
KTCM.PHIK	6.39
SBBR.KCHS	7.13
PHIK.KTCM	51.49
EGUN.KDOV	35.71
RJTY.KTCM	31.05
RKSO.KTCM	14.96
LICZ.KDOV	9.81
EDAF.LIPA	69.31
EDAF.OERY	100.03
KCHS.SKBO	64.34
KCHS.SBBR	26.89
KCHS.EDAF	3.91
KCHS.MNMG	0.22
EDAF.LIRN	1.84
EDAF.LGIR	10.46

KNGU.TISX	29.87
KSUU.PWAK	1.87
KSUU.OBBI	0.04
KSUU.OEDR	0.02
KTCM.RJTY	0.12
KWRI.EDAR	1.40
FHAW.KCOF	18.09
EGUN.KNGU	0.19
TJNR.KCHS	36.62
LTAG.LIPA	7.65
LTAG.LIRN	6.23
LTAG.LLBG	1.31
LTAG.LETO	2.13
PADK.PAED	2.33
PADK.PHIK	1.92
LIPA.EDAF	0.03
VTBD.RODN	2.76
WIIH.RODN	0.01
RODN.WIIH	1.96
RODN.FJDG	2.67
RODN.WSAP	10.55
PAED.PHIK	4.60
PHIK.PAED	2.95
PHIK.RJTY	7.57
PHIK.RKSO	3.02
PHIK.NSTU	1.84
RJOI.RJTY	6.99
RKJK.RODN	8.22

RKJK.RJTY	1.80
RKJK.RKSO	3.81
APLM.PHIK	0.35
RJSM.PHIK	0.19
FJDG.RODN	0.47
TJNR.TBPB	8.54
TJNR.MUGM	0.24
RJTY.VTBD	2.57
RJTY.EDAF	0.28
RJTY.RKPK	0.16
RJTY.RKTN	0.18
MKJP.MUGM	6.21
LIPA.LIRN	4.09
RKSO.PHIK	4.19
RKSO.RKPK	12.80
NSTU.PHIK	0.40
ASRI.ABAS	2.70
ASRI.APWR	6.88
EDAR.LPLA	20.44
EGUN.EDAF	18.93
EGUN.LPLA	0.09
BIKF.KNGU	2.86
EDAR.LIRN	6.83
EDAR.LICZ	16.59
EDAR.LGIR	1.34
LETO.LTAG	2.71
LETO.OBBI	0.23
LETO.LIRN	3.09

LETO.LERT	5.60
LETO.LICZ	1.95
LETO.LGIR	1.99
LERT.LPLA	0.42
LERT.LIRN	8.19
LICZ.OBBI	11.51
LICZ.OMFJ	1.22
LICZ.OEDR	1.98
LICZ.LETO	0.07
LICZ.LERT	11.36
LICZ.LTAG	4.82
PGUA.PHIK	4.93
PGUA.RJSM	0.01
PGUA.RJTY	2.83
PGUA.RKSO	9.66
LETO.LLBG	9.90
LERT.LTAG	5.51
EGUN.LICZ	5.85
EDAF.OJAF	5.00
KCHS.MGGT	17.36
LICZ.LIPA	11.99
LGIR.KDOV	19.61
KCHS.TAPA	18.67
KSUU.APLM	25.32
PADK.KSUU	29.37
SGAS.KCHS	7.16
PHIK.FJDG	9.79
SVMI.KCHS	5.55

APLM.PGUA	5.74
RJTY.PWAK	4.05
MSSS.KCHS	27.50
LLBG.KDOV	10.28
LTAG.LGIR	4.02
PAHT.PAED	4.53
OJAF.EDAR	6.34
OBBI.LICZ	9.34
RODN.PAED	6.68
PAED.RODN	7.99
PAED.RKSO	25.54
EDAF.EDAR	4.64
EDAF.LICZ	8.16
EDAF.LLBG	11.01
MPHO.MZBZ	9.79
RJTY.OMFJ	45.88
ASRI.PHIK	5.01
EDAR.EDAF	4.86
LERT.OBBI	4.52
LICZ.EDAF	4.03
LGIR.LIPA	7.53

KSUU.PGUA	4.29
KTCM.PADK	4.29
PADK.KTCM	4.29
RJTY.WSAP	16.5
RJTY.FJDG	4.29
RJTY.RKJK	4.29
RJTY.RKSO	13.5
RJTY.RJFF	4.29
RJTY.RJAW	4.29
RJTY.RJSM	13.5
RJTY.RJCB	4.29
RJTY.RJAM	4.29
PHIK.PJON	8.57
PHIK.PKWA	8.57
PHIK.PWAK	6.43
PHIK.PMDY	4.29
PGUA.KSUU	4.29
FJDG.RJTY	4.29
FJDG.OMFJ	12.86
OMFJ.FJDG	12.86
FJDG.WSAP	13.5
WSAP.FJDG	13.5
WSAP.RJTY	16.5
RKSO.RJTY	13.5
RKSO.RODN	13.5
RJFF.RJTY	4.29
RJAW.RJTY	4.29
RODN.RKPK	4.29

RODN.RKJK	13.5
RODN.RKSO	13.5
RODN.RJSM	13.5
RODN.PGUA	8.57
PGUA.RODN	8.57
RJSM.RJTY	13.5
RJSM.RKJK	13.5
RJSM.RODN	13.5
RJCB.RJTY	4.29
APLM.ASRI	4.29
ASRI.APLM	4.29
RJAM.RJTY	4.29
PJON.PHIK	8.57
PKWA.PHIK	8.57
PWAK.PHIK	6.43
PMDY.PHIK	4.29
RODN.VVNB	1.07
VVNB.RODN	1.07
KCHS.SBGL	2.14
SBGL.KCHS	2.14
KCHS.MHSC	4.29
KCHS.MPHO	4.29
KDOV.OJAF	2.14
KNGU.LIRN	12.86
KNGU.LICZ	15.5
KNGU.OBBI	12.86
KNGU.MUGM	12.86
KNGU.TJNR	4.29

KNGU.TXKF	3.21
KNGU.OMFJ	15.5
OMFJ.KNGU	15.5
KWRI.LPLA	13.5
KWRI.BGTL	4.29
EDAR.EGUN	12.86
EDAR.LETO	4.29
EDAR.HECA	4.29
EDAR.LIPA	8.57
EDAF.LETO	4.29
EDAF.OEDR	24.5
EDAF.LTAG	12.86
EGUN.EDAR	12.86
EGUN.LETO	4.29
EGUN.LIPA	4.29
LETO.EDAR	4.29
LETO.EDAF	4.29
LETO.EGUN	4.29
LETO.LIPA	4.29
LPLA.KWRI	13.5
LIPA.EDAR	8.57
LIPA.EGUN	4.29
LIPA.LETO	4.29
LIRN.KNGU	12.86
LIRN.LIEO	12.86
LIRN.LGSA	4.29
LGSA.LIRN	4.29
LIEO.LIRN	12.86

LIRN.LICZ	12.86
LICZ.KNGU	15.5
LICZ.LIRN	12.86
LTAG.EDAF	12.86
OJAF.KDOV	2.14
OBBI.KNGU	12.86
OEDR.EDAF	24.5
BGTL.KWRI	4.29
MHSC.KCHS	4.29
MHSC.MPHO	8.57
MPHO.KCHS	4.29
MPHO.MHSC	8.57
MUGM.KNGU	12.86
TJNR.KNGU	4.29
TJNR.MTPP	1.07
TJNR.TAPA	2.14
TAPA.TJNR	2.14
TJNR.TISX	4.29
TISX.TJNR	4.29
TXKF.KNGU	3.21
KWRI.CYYT	2.14
CYYT.KWRI	2.14
KCHS.SLLP	2.14
MPHO.SGAS	2.14
SGAS.MPHO	2.14
MPHO.SKBO	1.07
SKBO.MPHO	1.07
MPHO.SAEZ	2.14

SAEZ.MPHO	2.14
MPHO.MGGT	2.14
MGGT.MPHO	2.14
MPHO.SPIM	2.14
SPIM.MPHO	2.14
MPHO.SLLP	2.14
SLLP.MPHO	2.14
MPHO.MNMG	2.14
MPHO.SVMI	1.07
MPHO.SUMU	2.14
MPHO.MROC	2.14
MPHO.SBGL	2.14
MPHO.MSSS	8.57
MSSS.MPHO	8.57
MPHO.SCEL	2.14
SCEL.MPHO	2.14
MPHO.MHTG	8.57
MHTG.MPHO	8.57
MPHO.SEQU	1.07
SEQU.MPHO	1.07
MPHO.SBBR	2.14
HECA.EDAR	4.29
KCHS.SGAS	2.14
KCHS.SAEZ	2.14
SAEZ.KCHS	2.14
KCHS.FZAA	2.14
SLLP.KCHS	2.14
KCHS.SUMU	2.14

SUMU.KCHS	2.14
KCHS.FTTJ	1.07
KCHS.SCEL	2.14
SCEL.KCHS	2.14
MROC.MPHO	2.14
SBGL.MPHO	2.14
MNMG.MPHO	2.14
SVMI.MPHO	1.07
SUMU.MPHO	2.14
KCHS.GLRB	2.14
GLRB.KCHS	2.14

Appendix C. Application of Simple Point Kriging to the Residuals of Metamodel 1

In kriging, the first step is to determine the distance matrix, $\mathbf{h}$. Here, the distance matrix was obtained calculating the Euclidean distances between the design points. The Euclidean distance is given by

$$h_1 = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} \tag{C.1}$$

where x and y are the coordinate values of the design points. Recalling the first order 2^2 full factorial design of Metamodel 1 in Chapter 4, the following distance matrix is obtained:

$$\mathbf{h} = \begin{bmatrix} 0 & 2 & 2 & 2\sqrt{2} \\ 2 & 0 & 2\sqrt{2} & 2 \\ 2 & 2\sqrt{2} & 0 & 2 \\ 2\sqrt{2} & 2 & 2 & 0 \end{bmatrix}$$

Also, we can derive the distances of the points that are to be estimated to the other points. For example, the distance vector of the first validation point in the validation design (-0.5, -0.5) to the test design points is

$$\begin{bmatrix} 0.707 \\ 1.581 \\ 1.581 \\ 2.121 \end{bmatrix}$$

The second step is to construct the Equation (2.9) using a semi-veriogram function model. Here, the spherical model given in Equation (2.10) was used. The constant parameters are choosen as $C_0 = 0$, $C = 1.444 \times 10^{10}$ which is the variance between regression metamodel residuals, and $a = 2\sqrt{2}$ which is the maximum

distance in the distance matrix. By substituting the distance values into spherical model, for the fist validation point, Equation (2.9) becomes

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1.011\times10^{10} & 1.011\times10^{10} & 1.144\times10^{10} \\ 1 & 1.011\times10^{10} & 0 & 1.144\times10^{10} & 1.011\times10^{10} \\ 1 & 1.011\times10^{10} & 1.144\times10^{10} & 0 & 1.011\times10^{10} \\ 1 & 1.144\times10^{10} & 1.011\times10^{10} & 1.011\times10^{10} & 0 \end{bmatrix} \begin{bmatrix} \lambda \\ \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 4.201\times10^{9} \\ 8.594\times10^{9} \\ 8.594\times10^{9} \\ 1.046\times10^{10} \end{bmatrix}$$

The Lagrangian multiplier and the weights are calculated as

$$\begin{bmatrix} 4.725\times10^{7} \\ 0.596 \\ 0.178 \\ 0.178 \\ 0.048 \end{bmatrix}$$

Finally, substituting the weights into Equation (3.12), we find the kriged residual for the first validation design point as $\hat{\hat{\epsilon}}_1 = -26680$. The rest of the kriged residuals were found by following the same steps.

Appendix D. The Channel Cargo System of Air Mobility Command

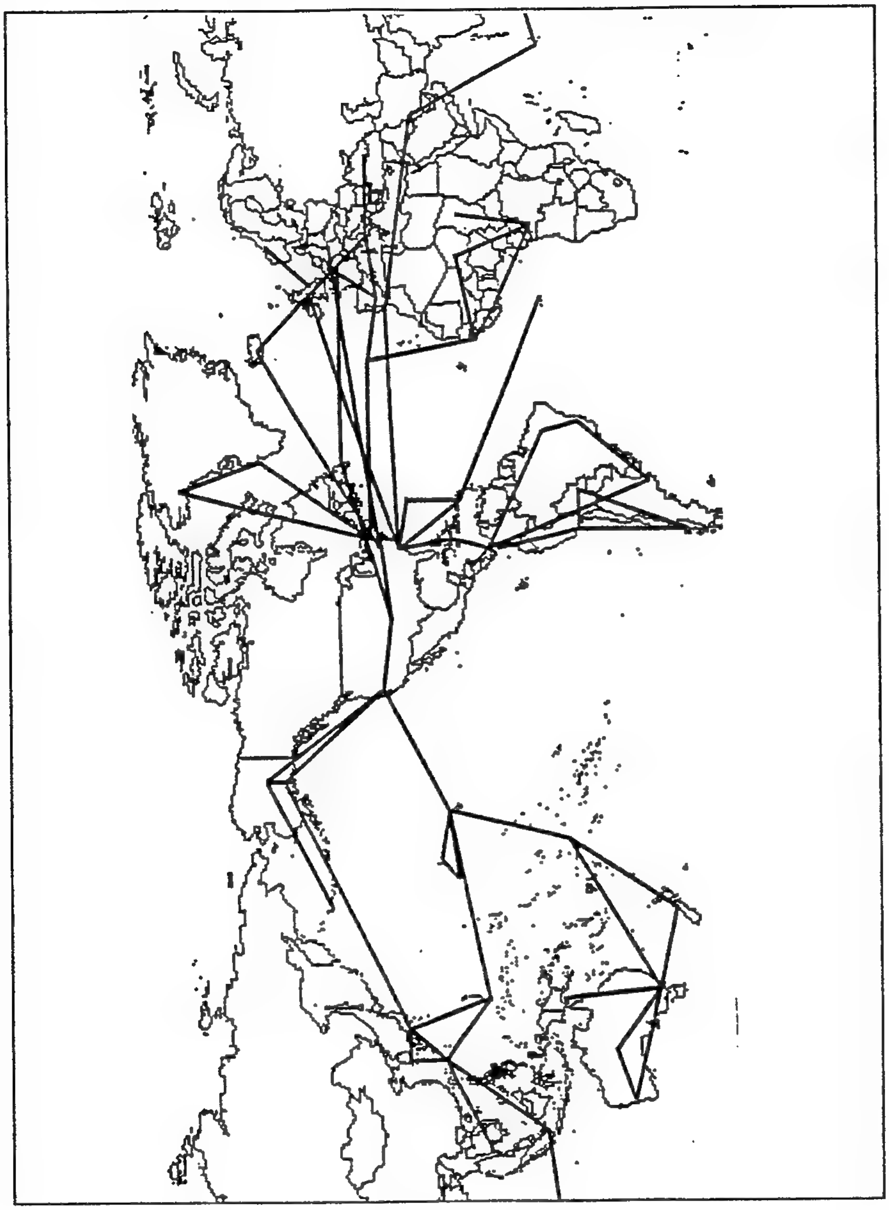

Figure D.1 World Map Showing Cargo Channels

Bibliography

1. Ackley, M., B. Carter J. Litko K. Ware and A. Whisman. *Decision Support for the Air Mobility Command Scheduled Cargo System*. Technical Report, HQ MAC/XPY Military Airlift Command, Scott AFB, Illinois 62225: Command Analysis Group, 1992.

2. Apostol, Tom M. *Mathmatical Analysis*. Reading, Massachusetts: Addision-Wesley Publishing Company, 1960.

3. Arsham, H. and M. Oblak. "Perturbation Analysis of General LP Models: A Unified Approach to Sensitivity Analysis, Parametric, Tolerance and More-for-less Analysis," *Mathematical and Computer Modelling*, *13*(8):70–102 (1990).

4. Bazaraa, M.S., J.J. Jarvis and Hanif D. Sherali. *Linear Programming and Network Flows* (Second edition Edition). New York: John Wiley and Sons, Inc.,, 1990.

5. Box, G.E.P. "The Exploration and Exploitation of Response Surfaces," *Biometrics*, *10*:485–493 (March 1954).

6. Box, G.E.P. and Norman Draper. *Empirical Model Building and Response Surfaces*. New York: John Wiley Sons, 1987.

7. Box, G.E.P. and Wilson K.B. "On the Experimental Attachment of Optimum Conditions," *Journal of the Royal Statistical Society*, *B*(13):1–45 (1951).

8. Bradley, Stephen P., Arnoldo C. Hax and Thomas L. Magnanti. *Applied Mathematical Programming*. Reading, Massachusetts: Addison-Wesley Publishing Company, 1977.

9. Brooke, A., Kendrick D. and Alexander Meeraus. *GAMS: User Guide, Release 2.25*. California: The Scientific Press, 1992.

10. Clark, Isobel. *Practical Geostatistics*. London, England: Applied Science Publishers, Ltd., 1979.

11. David, Michael. *Geostatistical Ore Reserve Estimation*. New York: Elsevier Scientific Publishing Company, 1977.

12. Davis, John C. *Statistics on Data Analysis in Geology* (Second edition Edition). New York: John Wiley Sons, 1986.

13. Fiacco, A.V. *Introduction to Sensitivity and Stability Analysis in Non-linear Programming*. New York: Academic Press, 1983.

14. Gal, Tomas and Josef Nedoma. "Multiparametric Linear Programming," *Management Science*, *18*(7):406–422 (March 1972).

15. Gass, S.I. and T.L. Saaty. "Parametric Objective Function (Part 2) Generalization," *Operation Research*, *3*(4):395–401 (November 1955).

16. Grant, Michael. *Analysis of Kriging Applied To Resolution Enhancement of Digital Satellite Imagery.* MS thesis, School of Engineering,Air Force Institute of Technology, Wright Patterson AFB, OH, December 1991.

17. Hadley, J. *Linear Programming (Second Printing).* Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1963.

18. Henley, Stephen. *Nonparametric Geostatistics.* New York: John Wiley Sons, 1981.

19. Hunter, J.S. and T.H. Naylor. "Experimental Designs for Computer Simulation Experiments," *Management Science*, *16*:422–434 (March 1970).

20. Isaaks, E.H. and R. Mohan Srivastasa. *An Introduction to Applied Geostatistics.* New York: Oxford University Press, 1989.

21. Johnson K.E., K.W. Bauer Jr., J.T. Moore and M. Grant. "Metamodelling Techniques in Multidimensional Optimality Analysis for Linear Programming." Report to AFIT/Department of Operational Sciences.

22. Journel, A. G. "Kriging in Terms of Projections," *Mathematical Geology*, *9*(6):563–586 (1977).

23. Journel, A. G. and Ch. J. Huijbregts. *Mining Geostatistics.* New York: Academic Press, 1978.

24. Karson, M.J., A.R. Manson and R.J. Hader. "Minimum Bias Estimation and Experimental Design for Response Surfaces," *Technometrics*, *11*:461–475 (August 1969).

25. Khuri, A.I. and J.A. Cornell. *Response Surfaces, Designs and Analysis.* Marcel Dekker Inc., ASQC Quality Press, 1987.

26. Lasdon, Leon S. *Optimization Theory for Large Systems.* New York: MacMillan Publishing Co., 1970.

27. Montgomery, D.C. *Design and Analysis of Experiments* (Third edition Edition). New York: John Wiley and Sons, Inc., 1991.

28. Mulvey, J. M., R.J. Vanderbei and S.A. Stavros. *Robust Optimization of Large Scale Systems.* Technical Report, Philadelphia, Pennsylvania: University of Pennsylvania, Dept. of Decision Sciences The Wharton School, 1993.

29. Murty, Katta G. *Linear Programming.* New York: John Wiley and Sons, 1983.

30. Neter, John et al. *Applied Linear Statistical Models* (Third edition Edition).

31. Smith, Palmer W. *A Methodology for Developing and Analyzing the Optimal Response of an Economic Criteria to Simultaneously Induced Events Using Input-Output, Mathematical Programming, and Response Surface Methodologies.* PhD dissertation, University of Alabama, July 1975.

32. Wagner, H.M. *N Parameter Sensitivity in Mathematical Programming Models.* Technical Report, Chapel Hill, North Carolina: University of North Carolina, 1992.

33. Ward, James E. and Richard E. Wendell. "Approaches to Sensitivity Analysis in Lenear Programming," *Annals of Operations Research*, *27*:3–38 (1990).

34. Wendell, Richard E. "Tolerance Approach to Sensitivity Analysis in Linear Programming," *Management Science*, *31*(5):564–578 (1985).

35. Winston, W.L. *Introduction to Mathematical Programming.* Boston: PWS-KENT Publishing Company, 1991.

Vita

1Lt. Osman İyde was born on 21 July 1968 in Adapazarı, Turkey. He graduated from Kuleli Military High School in İstanbul in 1987. After high school, he attended the Turkish Air Force Academy where he majored in Electronics. He completed his degree in 1991 and attended the Air Defense School in İzmir to become an intercept controller. After graduation in 1992, he was sent to the United States Air Force Institute of Technology for his MS degree in Operations Research. His next assignment will be in Turkish Air Force Headquarters in Ankara.

Permanent address: Atatürk Mahallesi,
Emekevler, G.O.Paşa Caddesi, No 48
81120 Kadıköy İstanbul
Türkiye

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
March 1995

3. REPORT TYPE AND DATES COVERED
Master's Thesis

4. TITLE AND SUBTITLE
RIGHT-HAND-SIDE MULTIDIMENSIONAL OPTIMALITY ANALYSIS OF A LARGE SCALE LINEAR PROGRAM USING METAMODELLING TECHNIQUES

5. FUNDING NUMBERS

6. AUTHOR(S)
Osman IYDE, 1Lt, TUAF

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Air Force Institute of Technology, WPAFB OH 45433-6583

8. PERFORMING ORGANIZATION REPORT NUMBER
AFIT/GOR/ENS/95M-13

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Air Mobility Command
Scott AFB, Illinois 62225

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

A methodology for optimality analysis of linear programs was developed by Johnson, Bauer, Moore, and Grant to create metamodels using response surface methodology techniques such as experimental design and least squares regression, and a geostatistical estimation technique, namely kriging. Metamodels have the form of a simple polynomial, and they predict the optimal objective function value of an LP for various levels of the constraints. They eliminate the necessity of determining which critical region contains the right-hand-side (RHS) vector of interest since they are valid over multiple critical regions.
The methodology of Johnson, *et al.* can be applied to large scale linear programming models. The developed metamodels of the large scale LP can provide some useful information about the relationships between the objective function value and the RHS vector of interest.

14. SUBJECT TERMS
Large Scale LPs,Optimality Analysis, Metamodels, RSM, Kriging

15. NUMBER OF PAGES
92

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

GENERAL INSTRUCTIONS FOR COMPLETING SF 298

The Report Documentation Page (RDP) is used in announcing and cataloging reports. It is important that this information be consistent with the rest of the report, particularly the cover and title page. Instructions for filling in each block of the form follow. It is important to ***stay within the lines*** to meet ***optical scanning requirements***.

Block 1. Agency Use Only *(Leave blank)*.

Block 2. Report Date. Full publication date including day, month, and year, if available (e.g. 1 Jan 88). Must cite at least the year.

Block 3. Type of Report and Dates Covered. State whether report is interim, final, etc. If applicable, enter inclusive report dates (e.g. 10 Jun 87 - 30 Jun 88).

Block 4. Title and Subtitle. A title is taken from the part of the report that provides the most meaningful and complete information. When a report is prepared in more than one volume, repeat the primary title, add volume number, and include subtitle for the specific volume. On classified documents enter the title classification in parentheses.

Block 5. Funding Numbers. To include contract and grant numbers; may include program element number(s), project number(s), task number(s), and work unit number(s). Use the following labels:

C - Contract
G - Grant
PE - Program Element
PR - Project
TA - Task
WU - Work Unit Accession No.

Block 6. Author(s). Name(s) of person(s) responsible for writing the report, performing the research, or credited with the content of the report. If editor or compiler, this should follow the name(s).

Block 7. Performing Organization Name(s) and Address(es). Self-explanatory.

Block 8. Performing Organization Report Number. Enter the unique alphanumeric report number(s) assigned by the organization performing the report.

Block 9. Sponsoring/Monitoring Agency Name(s) and Address(es). Self-explanatory.

Block 10. Sponsoring/Monitoring Agency Report Number. *(If known)*

Block 11. Supplementary Notes. Enter information not included elsewhere such as: Prepared in cooperation with...; Trans. of...; To be published in.... When a report is revised, include a statement whether the new report supersedes or supplements the older report.

Block 12a. Distribution/Availability Statement. Denotes public availability or limitations. Cite any availability to the public. Enter additional limitations or special markings in all capitals (e.g. NOFORN, REL, ITAR).

DOD - See DoDD 5230.24, "Distribution Statements on Technical Documents."
DOE - See authorities.
NASA - See Handbook NHB 2200.2.
NTIS - Leave blank.

Block 12b. Distribution Code.

DOD - Leave blank.
DOE - Enter DOE distribution categories from the Standard Distribution for Unclassified Scientific and Technical Reports.
NASA - Leave blank.
NTIS - Leave blank.

Block 13. Abstract. Include a brief *(Maximum 200 words)* factual summary of the most significant information contained in the report.

Block 14. Subject Terms. Keywords or phrases identifying major subjects in the report.

Block 15. Number of Pages. Enter the total number of pages.

Block 16. Price Code. Enter appropriate price code *(NTIS only)*.

Blocks 17. - 19. Security Classifications. Self-explanatory. Enter U.S. Security Classification in accordance with U.S. Security Regulations (i.e., UNCLASSIFIED). If form contains classified information, stamp classification on the top and bottom of the page.

Block 20. Limitation of Abstract. This block must be completed to assign a limitation to the abstract. Enter either UL (unlimited) or SAR (same as report). An entry in this block is necessary if the abstract is to be limited. If blank, the abstract is assumed to be unlimited.

★U.S.GPO:1990-0-273-271